Appendix B

Statement: the only ESS in the Norms game (assuming continuity and using Axelrod's parameters) is the state of total norm collapse ($b_i = 1$, $v_i = 0$ for all i).

Proof: Please bear in mind the definition of ESS and that eq. (3) and (4) must be fulfilled for the state to be an ESS. The following proves that the only state that satisfies all these conditions is $b_i = 1$, $v_i = 0$ for all i. All variables are assumed to be within the feasible range. We start by proving that a necessary condition for a state to be an ESS is that every agent is following the same strategy.

$$\mathrm{Exp}(Payoff_i) = \mathrm{Exp}(Payoff_j) \quad \forall\, i, j \in I\ (m \notin I);\ \forall\, m \in \Theta;\ \forall\, b_m, v_m$$

$$F = \mathrm{Exp}(Payoff_i) - \mathrm{Exp}(Payoff_j) = 0 \quad \forall\, i, j \in I;\ \forall\, m \in \Theta;\ \forall\, b_m, v_m$$

$$F = (T - H)\cdot(b_i - b_j) + \frac{E}{2}\left(v_i \left(b_m^2 + \sum_{\substack{k=1 \\ k \neq i,m}}^{n} b_k \right) - v_j \left(b_m^2 + \sum_{\substack{k=1 \\ k \neq j,m}}^{n} b_k \right) \right) +$$

$$+ \frac{P}{2}\left(b_i^2 \left(v_m + \sum_{\substack{k=1 \\ k \neq i,m}}^{n} v_k \right) - b_j^2 \left(v_m + \sum_{\substack{k=1 \\ k \neq j,m}}^{n} v_k \right) \right) = 0 \quad \forall\, i, j \in I;\ \forall\, m \in \Theta;\ \forall\, b_m, v_m$$

$$\frac{\partial F}{\partial b_m} = 0 \quad \forall\, i, j \in I;\ \forall\, m \in \Theta;\ \forall\, b_m \in (0,1)$$

$$E \cdot b_m \cdot (v_i - v_j) = 0 \quad \forall\, i, j \in I;\ \forall\, m \in \Theta;\ \forall\, b_m \in (0,1)$$

$$v_i = v_j \quad \forall\, i, j \in I;\ \forall\, m \in \Theta \ \Rightarrow\ v_i = v_j \quad \forall\, i, j \in \Theta$$

$$\frac{\partial F}{\partial v_m} = 0 \quad \forall\, i, j \in I;\ \forall\, m \in \Theta;\ \forall\, v_m \in (0,1)$$

$$\frac{P}{2}\cdot(b_i^2 - b_j^2) = 0 \quad \forall\, i, j \in I;\ \forall\, m \in \Theta$$

$$b_i = b_j \quad \forall\, i, j \in I;\ \forall\, m \in \Theta \ \Rightarrow\ b_i = b_j \quad \forall\, i, j \in \Theta$$

Thus we have proved that a necessary condition for ESS is that every agent has the same strategy. We assume from now on that $b_i = B \quad v_i = V \quad \forall\, i \in \Theta$

$$\frac{\partial \mathrm{Exp}(Payoff_m)}{\partial b_m} = T + (n-1)\cdot B \cdot V \cdot P \qquad \frac{\partial \mathrm{Exp}(Payoff_i)}{\partial b_m} = H + E \cdot V \cdot B \quad i \neq m$$

$$\frac{\partial \mathrm{Exp}(Payoff_m)}{\partial v_m} = \frac{E}{2}(n-1)\cdot B^2 \qquad \frac{\partial \mathrm{Exp}(Payoff_i)}{\partial v_m} = \frac{P}{2} B^2 \quad i \neq m$$

$$\left.\frac{\partial \mathrm{Exp}(Payoff_m)}{\partial b_m}\right|_{B=0} > \left.\frac{\partial \mathrm{Exp}(Payoff_i)}{\partial b_m}\right|_{B=0} \Rightarrow \{\text{eq. (3)}\} \Rightarrow \quad B \neq 0$$

$$\frac{\partial \mathrm{Exp}(Payoff_m)}{\partial v_m} = \frac{E}{2}(n-1)\cdot B^2 < \frac{P}{2} B^2 = \frac{\partial \mathrm{Exp}(Payoff_i)}{\partial v_m} \quad \forall\, B \neq 0 \Rightarrow \{\text{eq. (4)}\} \Rightarrow V = 0$$

$$\left.\frac{\partial \mathrm{Exp}(Payoff_m)}{\partial b_m}\right|_{V=0} > \left.\frac{\partial \mathrm{Exp}(Payoff_i)}{\partial b_m}\right|_{V=0} \Rightarrow \{\text{eq. (3)}\} \Rightarrow \quad B = 1$$

Therefore it is proved that ($b_i = 1$, $v_i = 0$ for all i) is a necessary condition for ESS in the Norms game. Now we prove that it is sufficient. Let m be a potential mutant agent.

$$\mathrm{Exp}(Payoff_m) = b_m T + (n-1)H + E\frac{v_m}{2}(n-1) \qquad b_i = 1,\ v_i = 0,\ \forall\, i \neq m$$

$$\mathrm{Exp}(\mathit{Payoff}_i) = T + (b_m + n - 2)H + \frac{v_m}{2}P \qquad \forall\, i \neq m;\;\; b_i = 1,\; v_i = 0, \forall\, i \neq m$$

$$\mathrm{Exp}(\mathit{Payoff}_m) < \mathrm{Exp}(\mathit{Payoff}_i) \qquad \forall\, b_m, v_m \;\; \left(b_m \neq 1 \text{ OR } v_m \neq 0\right);\;\; b_i = 1,\; v_i = 0, \forall\, i \neq m$$