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Modelling Opinion Formation with Physics Tools: Call for Closer Link with Reality

Abstract

The growing field of studies of opinion formation using physical formalisms and computer simulation based tools suffers from relative lack of connection to the 'real world' societal behaviour. Such sociophysics research should aim at explaining observations or at proposing new ones. Unfortunately, this is not always the case, as many works concentrate more on the models themselves than on the social phenomena. Moreover, the simplifications proposed in simulations often sacrifice realism on the altar of computability. There are several ways to improve the value of the research, the most important by promoting truly multidisciplinary cooperation between physicists aiming to describe social phenomena and sociologists studying the phenomena in the field. In the specific case of modelling of opinion formation there are a few technical ideas which might bring the computer models much closer to reality, and therefore to improve the predictive value of the sociophysics approach.

Keywords:

Opinion modelling - generalities

1.1

One of the major problems with 'social physics' or sociophysics literature, especially the exploration and understanding of social processes by means of computer simulation is the lack of connection to real life examples. In an early work in the field of opinion modelling ([Nowak 1996](#)) we find the following hope: 'Proponents of simulations view them as a powerful tool that will transform social sciences by bringing precision and rigour, into social theories'. Yet, despite the undoubted advances in simulation techniques and computer power, the transformation has not happened. Partially because modelling has became largely a self-sufficient exercise. In a recent Forum article in *Journal of Artificial Societies and Social Simulation* ([Epstein 2008](#)), the author lists seventeen reasons for using models, the three topmost being prediction, explanation and guiding data collection. All of these goals are related to real data and phenomena. Moss and Edmonds ([2005](#)) conclude their paper, titled *Towards Good Social Science*, with a list of pros and cons of the practice, with the advice for the good works starting with 'the fundamental priority of observation and evidence over models and theory', and the list of malpractices which starts with 'the unthinking and inappropriate use of analytic and statistical techniques'. In the years that have passed since this publication not a lot has changed.

1.2

This commentary is prompted by the generalisation of observations made while reviewing publications, which I am asked to do from time to time for several journals. I am privileged to be in a position of an 'interested outsider' to the field, no longer affiliated with any research institution, without the pressure of publication and career. My opinion may then be classified as a voice from the civil society, outside of Academia. Of course I lack the expert knowledge of all the developments in the field, so some of the active authors might consider my remarks unjust. I apologize to all those who do indeed combine, in sociophysics studies, the genuine social data with a resolve to provide an explanation through similarity with physical systems or through computer models. Unfortunately, too many papers written on opinion or consensus modelling (to stick with the branch of sociophysics I am the most familiar with) are totally decoupled from reality. The forthcoming review of the field ([Castellano 2007](#)), which presents quite detailed view of the models and theories shows how relatively limited are the

attempts to compare the simulations and observations. In my personal experience the reactions to referee calls for closer connection with observed phenomena are also quite disheartening. In one case, the authors have openly stated that they were interested in the results of simulations as such, not with their link to reality. They have even changed the text of the paper, striking out the original token references to social phenomena, and leaving only blatant statement 'we study the model because it is an interesting model'.

1.3

Experience from the history of physics teaches us that the interplay between experiment (or observations) and theoretical models is crucial factor for success. To preserve the value that the 'physics' part adds to social studies, a return to this traditional experiment–theory balance is needed. There are a few ways to achieve this, some simple, some not so simple. The foremost is to strive for real multidisciplinary effort. Involvement of both physicists (knowledgeable about statistical mechanics, computer simulations etc.) and sociologists (with the knowledge about real situations where our understanding would benefit from the numerical models) in joint research teams is not always easy, but it seems necessary to keep the discipline alive and in touch with reality. In many cases it would also be fruitful to include biologists or neurologists into the teams. They would provide another perspective on these aspects of phenomena that are related to our biological functions. Such links may be very illuminative and sometimes quite unexpected to researchers outside the field. Just to point out an example: a recent study of Williams ([2008](#)) shows that there is direct, neurologically founded link between the physical warmth/coldness and the way people act towards others, the psychological feelings of warmth and coldness. Such studies show that sometimes it is necessary to broaden the outlook and the team as much as possible.

1.4

Unfortunately, in many cases the physicists are so fascinated with their tools that they forget to ask if the topic they choose to study—and the details of the model they use—are indeed interesting to the social scientists. As in 'traditional' physics, observations of societies in action, which are here the equivalent of experimental physics, are the ultimate litmus paper for the models and theories.

1.5

Of course, it is easier said than done. The cost and difficulty of making real social experiments or large scale observations is enormous, especially compared to computer programming. It is much faster to pick one of the fashionable topics, do some Monte Carlo simulations and write a paper about the results. Which is an important factor, especially in the age of 'publish or perish', and career based on the number of publications.

1.6

What do the such works concentrate upon? Quite often it is a derivation of an analytical formula for a particular simplified system, often borrowing some names from human society roles and functions to suggest direct applicability. The (ab)use of mathematical formulae in social sciences is still thought to be of supreme value, despite being rightly ridiculed for more than half century ([Parkinson 1958](#); [Andreski 1972](#)). When there are no simple analytical expressions, the topic is often some nice observation of opinion changes, contrasting exponential decay versus fat tail distribution, or some other feature persistent in Monte Carlo simulations. In choosing which results are interesting, sociophysicists are often guided by folk psychology, and select as interesting those that agree with their own general expectations. Without checking if they are bringing any new insights to social sciences or not. Or even if their intuitions (and results) are supported by real observations, which are often at odds with naive expectations. I know, I have been guilty of the sin myself, and I have seen it numerous times in the literature.

1.7

When we see our simulations producing nice curves (especially ones that we can approximate by analytical formulas), when we see others publishing similar output of *their* computer programs, the imperative to publish becomes very strong. The main question ceases to be 'am I describing a real phenomenon correctly and in sufficient detail? ', and turns to preparing the arguments to answer the eventual question of the journal Referee: 'does the paper contain enough new physics to be published in XXX? '

1.8

Despite the possible unpopularity of the idea in the sociophysicist community, I would go as far as to postulate that the leading role in such collaborations should go to the sociologists or psychologists. *They* are at the forefront of the research, studying the real life phenomena. Physicists provide the tools, and, at times, interesting insights and working models, but their role is clearly subservient. After all, nobody (I hope) believes that humans are mindless cellular automata governed by a few simple rules and characterised by a few numeric parameters. Such simplification might be useful in certain circumstances but to put it above the behaviour of real people in real situations is preposterous. Modelling may be very valuable, especially when it provides new understanding, new insights, points out new questions and directions for research. But it should always remain modelling of some real situation or process, and it is this process or situation which is the ultimate goal of activity.

🌍Technical improvements

2.1

In addition to the need for better ties with observational data and promoting truly multidisciplinary way of conducting research, there are as well some technical aspects of the modelling approach that could adapt them better to realistic conditions. A good example of a branch of sociophysics that would greatly benefit from rather simple changes is the study of opinion formation. The field has a long tradition of studies using methods derived from physics. A few of the works, coming from author's rather small collection are listed in the Appendix, where some characteristics are summarized.

2.2

Despite this limitation, there are some specific examples where the approaches could be improved. They may be grouped into two classes:

- improvement of the modelling quality of the network of links between the interacting agents, leading to global processes in achieving consensus or maintaining dispersed opinions;
- improvement of the description of these agents, their interactions and the individual opinion change process as such.

Both groups offer relatively straightforward ways of improving current state of research.

Social network modelling

2.3

Let's consider network interactions first. The early approaches have used very simplistic connection geometry, usually square lattice with nearest neighbour interactions. Keeping such geometry was fruitful in in the segregation model ([Schelling 1971](#)), because of its direct link with physical, 1D or 2D neighbourhood, and computationally justified in early works (e.g. [Nowak 1990](#)). With the introduction of small world idea ([Milgram 1967](#)) and associated shortcut network topologies ([Watts 1998](#); [Watts 1999](#); [Strogatz 2001](#)) a new 'science of the networks' has been born. This has resulted in enormously rich literature (some examples of review papers are [Albert 2002](#); [Barabasi 2001](#); [Dorogovtsev 2002](#); [Dorogovtsev 2003](#), [Newman 2000](#); [Newman 2003](#)). The advantage of the new network models (Strogatz–Watts, small world, scale free etc.) is that they represent much better the actual relationships between people and groups.

2.4

Many of the new papers on the spread of opinions in populations do take into account these non-trivial connection topologies, and have led to interesting results. For example, the existence of small-world type shortcuts allows information to spread quickly through the society. Local opinions are capable of influencing distant parts immediately. In scale free networks the hubs—highly connected individuals—can influence a lot of other members of society. The ease or difficulty of convincing such highly connected agents is often crucial to achieving consensus. Another aspect brought in by the new generation of models is due to non-uniform nature of connections, which allows more variability in describing local environments. In summary, these simulations may be much closer to the real world.

2.5

Yet while the step from simple, geometric approaches to a networked one seems obvious, there are still quite a few papers that use the inflexible, nearest neighbour connection scheme. This is probably due to the simplicity of encoding the 2D geometry into computer models—compute power was an issue twenty years ago, but certainly not today. In my opinion, any research aiming at description of complex human societies should opt for these more advanced interconnection topologies. The use of 'flat earth' can be justified only in cases, where for real reasons, the actual interactions take such form. While rare, such situations do exist and might be worth studying: for example the spread of emotional states and spiteful or cooperative behaviour in traffic jams, where each driver sees and reacts only to the nearest, visible neighbours.

2.6

The improvement brought by structurally rich networks is already enormous. The next step is to change the view from static to dynamical. In our societies, connections are continuously created, change their strength, sometimes they are abruptly severed. The models should include such time dependent phenomena. Consider, for example, how often the reaction to an encounter with a person of opposite opinion is to cut off the links to that person, without any of the participants changing their opinions. This process has much deeper consequences than the typical actions of averaging of the different opinions or forced conviction, decided by relative strengths of the agents. The adjustments of the network topology, connectedness between subgroups, flow of information through the network can profoundly change the results. It may turn that some effects would be far easier to explain within dynamic networks than when the model uses dynamic agents in a static network. Of course some links can not be broken (family and workplace relationships, for example). But many others can and are. Supporters of different political parties rarely indulge in an effort to convince each other. Much more often they tend to close the circle of friends and acquaintances, to talk to people who support their views.

2.7

One could expect that models with dynamic, reacting networks would show interesting effects in studies of persistence of minority or extreme opinions; preserved not because of particular strength of the believers but because of their tendency to sever as many links with majority as possible. Such systems with dynamically changing connectivity of the network were considered by Gil (2006) and Zanette (2006), however the agent properties and behaviours in their model were very simple.

2.8

Time-dependent topologies provide quite interesting way to describe medium-sized groups, where statistical methods are less applicable. At the same time, sociological observations and experiments is more feasible for such small and mid-sized groups. It might be interesting to see if the predictions of agent-based models would describe better the stability of minority groups and provide suggestions as to the methods and outcomes of their re-integration into the main society, e.g. by enhancing the links that are, by their nature, less affected by opinion differences. Such approaches are historically known to be quite effective, in almost all human societies (for example in forms of pacifying bonds forged by intergroup marriages). Yet recent history provides examples that such links might be insufficient if other factors are simply too strong. The ethnic conflicts that erupted after the disintegration of Yugoslavia, despite many years of extensive interlinking between communities during Tito's years can be a very interesting ground for modelling studies. Another field where suitable and interesting data could be used might be in time-flow communities, such as schools, where easily recognized and monitored imposed links (such as division into classes and age groups) intertwine with temporary coalitions. Experiments with consensus formation and information acceptance could provide a good comparison for various theoretical models and suggestions for further development.

2.9

There is an interesting parallel between such dynamic social networks and consensus studies and certain models of neural correlates of psychic phenomena. In some approaches, the time dependent, variable 'coalitions' of neurons, excited together and corresponding to mental states are vying for the dominance in overall brain activity (see, e.g. Chialvo 1999; Chialvo 2004; Sporns 2000; John 2001). This similarity may, perhaps, be quite stimulating in terms of concepts and methods.

Interaction modelling

2.10

Second group of proposed improvements relates to the description of the agents, their characteristics and interactions. In modelling human societies we are faced with ostensibly insurmountable task of compressing the multifaceted human nature, driven by individual histories and almost infinitely rich in motivations and action choices by a simple mathematical or programming entity, with a limited repertoire of actions and very basic description. The justification for such simplification comes, fortunately, not from limitation of sociophysics, but from empirical evidence from psychology and sociology. Yes, in certain circumstances one can expect people to react in predictable, rather simple ways. Thus, it is possible that a study using computer based agents would yield sensible and valuable results. The crucial questions become: in which situations can we use such simplifications and how to choose which parameters should describe the agents and their interactions? In particular, what should be the set of descriptors needed in studies of opinion changes?

- In some models there is but one parameter: opinion held by the agent. Then the whole process of interaction between agents can depend only on the similarity or difference between their views. Despite the obvious oversimplification of such approach, there are examples in the current literature that use it. Typically, the outcome of an encounter is a change of opinion in one of the agents. In the simplest, discrete models, opinion may take one of the +1 or -1 values and change of opinion is similar to a spin flip. The probability of the opinion flip is given by the values of opinions in the neighbourhood of the agent, where the definition of the 'neighbourhood' depends on the network topology.
- In continuous models, opinions are usually bound between +1 and -1, and the change can take a simple weighted averaging of opinions, for example when an encounter of two agents, holding opinions o_i and o_j, would lead to them both having somewhat more similar opinions o'_i and o'_j, somewhere 'in between' the initial values. Such continuous averaging would eventually cause all opinions to collapse to some medium value, dictated by external parameters, were it not for the extra measures that are aimed at mimicking more 'human-like' behaviour. A good example is given by assuming that if the opinions are sufficiently disparate, there is no chance of averaging. The models can then lead to persistence of more than one opinion.

  Models based on single parameter have the advantage of being simple to analyse, but reproducing complex agent behaviours (when to change one's own opinion, when to refuse, how strong is the influence etc.) via a single number is rather difficult and range of options is limited.

- In more advanced approaches there are two agent characteristics—its opinion (which may be continuous or discrete) and its and strength of its ability to convince other agents and resilience to other's opinions. This allows more flexibility in describing the

interaction process: the influences between agents are decoupled from their opinions, which allows to assign different 'social roles' and 'powers' to them, reflecting social differences.

- The above model may be further expanded by adding the strengths of the links connecting pairs of agents. This might correspond to physical or psychological 'closeness' of the agents, or functional/professional relationships. In such situation, even a strong and convincing individual would have less impact on agents that are only weakly connected. Technically, the strength of links could be put into the simulations directly, or it can be calculated from the 'network distance' between a pair of agents.
- Another direction of increasing complexity of the model is by replacing a single opinion by a set of opinions (worldview). The process of achieving consensus would result from multidimensional optimization. This makes the model much more realistic, as the opinions of people frequently form very connected set, where an opinion on a single issue might depend on other issues, and ability to communicate and influence would depend on many of such factors. Of course, going from single parameter opinion to, say, an n–dimensional vector may be further accompanied by including the agent and link strengths.

2.11

As we see, the choice of models is already quite rich. Moreover, in addition to agent characteristics, we should also consider variations of the interactions process. What changes during an encounter—is it only agent's opinion? For example, an encounter may lead to increase or decrease of its strength, susceptibility or persuasiveness. How many steps are in the process leading to individual opinion change? Should the process be broken into steps of information exchange, evaluation and positioning? Are there any thresholds (similar to neuron excitation threshold) that trigger any change? How should the Monte Carlo simulations represent human interactions, what is the relation between 'computer time' and reality? Do we really meet each other thousands of times that seem to be needed to achieve stability in computer experiments? Or should the real processes of opinion formation, where we meet just a few people just a few times, be described using transient, nonequilibrium states? The last question is particularly important, as these transients are the very parts of simulations that nobody wants to talk about, because they depend on particular initial conditions. But this is exactly the issue that makes social sciences interesting and difficult—how to distinguish between universal and historical view. In reality a lot depends on the initial conditions and historical accidents.

2.12

It seems that one of the interesting and almost untouched ideas is to take into account the economics of changing the opinions of others. Simulations of cooperation and aggression in societies have used successfully the pay off and cost of attitude detection or cost of retribution for a long time. Thus the step is rather natural and obvious for consensus formation—yet it is seldom used. Including such cost calculation in simulations is rather straightforward, and easily incorporated in MC procedures. Such considerations would be especially important in modelling directed changes, i.e. changes brought by individual leaders and organizations. Measuring the effect of different strategies for using available resources to achieve the desired level of consensus would make sociophysical studies much more interesting and close to reality of commercial advertisement or political propaganda.

2.13

It should be noted that above certain limit of complexity the models cease to be useful, because it is just as difficult to understand them as it is in real social situations. The balance between the number of parameters that can be varied in a model on one hand, and, on the other, its resemblance to reality and predictive power must be preserved. And the decisive factor would not be the preference for this or that model, for example continuous vs. discrete variables, but comparison with observations. Making the models more and more complex just for the sake of adding one more parameter and making the paper fit for publication does not seem in the spirit of physics any more. On the other hand, it should be noted that even complex models can be analysed in more depth than real social situations. For example, by meticulous checking of the role of parameters through repeated simulations—something that is not possible in real life. However, from the point of view of an outside observer I would much prefer simpler models, but ones aimed at explaining a behaviour of a real system, where the meaning of parameters can be understood. A perfect—and very funny—example is provided by the analysis of scientific publications, with the observation that, quite probably, a lot of scientists do not read the papers they cite. This real world phenomenon has allowed to propose a very simple computer model of 'randomly citing scientist', which has reproduced, remarkably closely the actual distribution of frequency of citings (Simkin 2003; Simkin 2005a; Simkin 2005b; Simkin 2006). The combination is an almost perfect example of what sociophysics should be: direct link to real world, a simple model where the role of parameters is understood, close resemblance between simulation results and observations. Thus, it is possible to go beyond the ghetto of formulae and Monte Carlo runs. Simkin's model, simple as it is, has given a valuable insight into the observed behaviour. This proves that social modeling is most useful in analyses of repeatable phenomena and their statistical properties, not in individual events.

2.14

Returning to opinion formation studies and the growing number of models: only empirical verification can allow to decide which model is the most appropriate for which situation. The

brief review, assembled in the Appendix, shows that only a small fraction of the works even attempt to look at the reality. So much more praise should go for the authors who take this important and difficult step. But for the others, I think that a change in overall atmosphere is needed, even though it might be considered hurtful for existing research projects. The journal editors and referees should stress the need to go beyond studying models for model's sake each time the consider publishing a manuscript; funding agencies should promote truly multidisciplinary cooperation between different groups of researchers. The success of such cooperation in other applications of physical models to social phenomena (for example to understanding traffic patterns) shows that it can be done, and that physics can be valuable tool to expand our understanding of humanity. At the same time the recent financial crisis has shown that coupling of physics–derived financial models with human greed and other 'un–mathematical' behaviours does not lead to predictable results, teaching us more caution in blind belief in statistical tools and models. The same caution should be used for opinion studies.

Appendix

A.1

In the Appendix, I have aimed at synthetic presentation of the characteristics of a small selected subset of the literature on the physical modelling of opinion formation. The list of papers used for this analysis is not meant to be exhaustive, as the present author is only an amateur with limited view of the field, and inclusion or omission of particular paper does not represent any judgement on the value.

A.2

There are several 'mainstream' models used in modelling consensus formation. Among the most widely used are Sznajd model, Deffuant model, Krause–Hegelsmann and other models (Sznajd 2000; Deffuant 2000; Hegelsmann 2002). Below is a very short summary of the main models.

Sznajd model
considers a population of agents with discrete (+1/-1) opinions o_i. The dynamics of opinions in the model is based on an influence of pair of neighbouring agents, with two basic rules: 'ferromagnetic' (if the opinions of agents o_i and o_{i+1} are the same then opinions of neighbouring agents o_{i-1} and o_{i+2} will be changed to o_i) and 'antiferromagnetic' (when the opinions o_i and o_{i+1} are different then an antiferromagnetic pattern forms, $o_{i-1} = - o_i = o_{i+1} = - o_{i+2}$). The model has been originally proposed for 1D geometry, later extended to 2D and more complex network topologies. Some variants use only the 'ferromagnetic' part of the interaction, assuming that if the opinions of the starting pair of agents differ, they have no influence on their neighbours. In Sznajd model, the opinion flows out from a group of agreeing agents, rather than in, from the environment to an agent, but in some cases the results are similar.

Deffuant model
uses a population of agents with continuous, bounded range of opinions. At each time step any two randomly chosen agents interact (global connectivity). They re–adjust their opinions when their difference of opinion is smaller in magnitude than a threshold. When the two agents have opinions o_i and o_j then if the difference of opinions is small enough, $| o_i - o_j | < d$, after the encounter their opinions would be, respectively $o'_i = o_i + \mu (o_j - o_i)$ and $o'_j = o_j + \mu (o_i - o_j)$, where μ is the convergence parameter taken between 0 and 0.5 during the simulations.

Krause–Hegselmann
The strength of interactions between agents is given by an array of parameters s_{ij}, so that at every interaction step the agent's opinion is modified to $o'_i = g_i o_i + (1 - g_i) \Sigma_j (s_{ij} o_j)$, where g_i is the agent's susceptibility. The interaction here may be considered global, or, if proper limitations of the strength of influence matrix are imposed, it may represent any type of the network. In the original papers a stochastic form of this matrix has been used, with normalisation condition .

There are quite a few variants and combinations of the main models mentioned above. The opinions might be multidimensional, and the processes of individual opinion change due to an encounter or influence of other agents might be based on the interplay between various 'interest points' (dimensions of the opinion vector). In addition to the strength of the influence some other characteristics of an agent have been proposed, for example 'susceptibility' and 'persuasiveness'. Some papers consider the role of special agents, for example extremists (agents with unchangeable opinions, in the continuum models usually located at the far ends of the spectrum of opinions), or leaders (agents 'equipped' with particularly large values of strength or otherwise capable of playing a crucial role). Some models add general biases (advertising influence) to explain the reaction of the society to external influences.

A.3

The network aspect of the models is also very differentiated. Starting from simple Euclidean geometry (usually discretized) in one, two or more dimensions, through globally connected meshes to very advanced network topologies. Then the links may have special attributes,

influencing the interactions between agents. In some models all encounters, when they are allowed are in principle uniform. In other models, the 'social distance', modelled by network properties changes the outcome of encounters.

A.4

The variety of the models shows how active is the research field, but, at the same time there is no consensus as to which model is 'the best'. It is quite natural, as most likely various models would offer advantages for different social phenomena. Only comparison with social, empirical data would enable us to judge if adding further refinements is necessary or not. But such comparison seems to be quite rare. The following Table attempts to characterise the a small subset of publications related to the issues discussed in the main part of the paper. Perhaps such summary would be useful in showing a bird's eye view of the techniques used and possibilities to improve the methods. For a recent review and richer literature subset see Castellano ([2007](#)).

Table 1: Selected examples of opinion modelling works

Reference	Main topic	Topology	Opinion model	Agent characteristics	Link characteristics	Special features	Comparison with real data
[Behera 2003](#)	comparison of Sznajd and Voter models	1D	Sznajd model, Voter model	opinion (discrete)	NN, NNN, uniform strength	influence of general bias; synchronous vs. asynchronous update	none
[Bernardes 2001](#)	scaling of the cluster growth	2D	Sznajd model	opinion (discrete)	uniform	modified Sznajd model used to compare with empirical data; analysis applied in transient regime	yes, Brazilian elections
[Bernardes 2002](#)	explanation of election results and the Sznajd model on Barabasi network	3D cubic; scale free	Sznajd model	opinion (discrete)	uniform	comparing tempo of equilibrium in 3D and network topologies	yes, Brazilian elections
[Caruso 2005](#)	Multidimensional analysis of contexts for decisions with local agent interactions but also with effects of global factors	global connectivity	bipolar, with number of agents of fixed opinion (RC,LC) and undecided (CG)	payoff based on multidimensional context vector	global, with strengths of interaction dependent on type of agent	some agents form coalitions that do not change their opinion; general bias influence	yes, Italian and German elections
[Conradt 2003](#)	decision making in animals	global connectivity	binary	based on costs of synchronization of actions, also with incomplete information	global, uniform strength	contrast between `despotic' and `democratic' groups	yes, many animal groups, some references to human societies
[Conradt 2005](#)	review of consensus decision making in animals	many models mentioned in a review	many examples of animal behaviour				
[Deffuant 2000](#)	presentation of consensus model with continuous opinion spectrum	global connectivity; 2D square	continuous, adjusted if opinions are closer than certain threshold	opinion, threshold for agreement	uniform strength; global connectivity or NN compared	discussion of multidimensional opinions (vector)	none
[Deffuant 2002](#)	consensus model with analysis of uncertainty	global	continuous, adjusted if opinions are closer than certain threshold	opinion, threshold for agreement, uncertainty	uniform strength	analysis of extreme opinions survival	none
[Deffuant 2004](#)	modelling opinion shift to extreme values	global	continuous, adjusted if opinions are closer than certain	opinion, threshold for agreement, uncertainty	uniform strength	special agents (extremists) with opinions located at edges of the spectrum and	some reference to psychological data, no comparison

			threshold			low uncertainty are used	
[Dyer 2008](#)	consensus in human crowds	no simulation model introduced	experiments on crowd behaviour				
[Elgazzar 2002](#)	opinion formation on small-world network	small world	Sznajd model	opinion	NN and shortcuts, uniform strength		none
[Fortunato 2004b](#)	network universality of confidence parameter dependence	global, 2D lattice, random network, scale free network	Deffuant model, continuous	opinion, threshold for agreement	uniform	comparison of network topologies	none
[Fortunato 2004a](#)	combination of Sznajd and Deffuant models	2D contrasted with scale-free	modified Sznajd model	opinions, continuous	NN	weak and strong versions of opinion change are discusses	none
[Fortunato 2005](#)	comparison of Krause-Hegselmann, Sznajd and Deffuant models	scale free	various versions of characteristics and dynamics, corresponding to the models being compared	reactions to extreme events: external modification of single agent's opinion and subsequent dynamics	none		
[Fortunato 2007](#)	simple `word-of-mouth' influence model used to explain election results universality	hierarchical network	unidirectional influence	candidate preference	hierarchical, unidirectional	discovery of universal features in actual election results and simple fit with `word-of-mouth' model	yes, many examples of electoral results
[Galam 1997](#)	achieving consensus through minimization of individual conflicts	Ising-like model; agents interact with a fixed fraction of all agents	statistical analytic considerations, no simulation; agents characterised by opinion, discrete; social pressure; internal preference		none		
[Gil 2006](#)	opinion formation on evolving networks	initially global; as results of interactions links are cut, topology changes	opinions adjusted with probability p1, links cut with probability p2	opinion (binary)	uniform	formation of separated clusters	none
[Hegelsmann 2002](#)	introduction of Krause-Hegelsmann model,	global	Krause-Hegelsmann	opinion, time dependent influence matrix, susceptibility	weighted interactions with other agents		none
[Holyst 2001](#)	social impact of strong leaders	2D	Nowak-Latane model, with strong leaders	opinion, strength of influence, external influence, social `temperature'	weighted interactions with other agents, strength decreasing with physical distance	comparison between simulations and analytical `mean-field' approximations	none
[Kacperski 1999](#)	social impact of strong leaders	2D	Nowak-Latane model, with strong leaders	opinion, strength of influence, external influence, social `temperature'	weighted interactions with other agents, strength decreasing with physical distance	study of effects of flips of leader's opinions	none
[Kacperski 2000](#)	phase transitions of	global	Nowak-Latane model, with	opinion, strength of influence,	weighted interactions	presence of universal `phase	none

	global opinions for groups with leaders		strong leaders		external influence, social `temperature'	with other agents, strength determined by arbitrary `adjacency matrix'	transitions'	
Lewenstein 1992	mean field theory of social impact	comparison of global interactions, sparse network, hierarchical network, 2D lattice		modified Nowak–Latane model	opinion (binary); persuasiveness; supportiveness	interaction strength weighted by `social distance'	comparison of various geometries; analysis of effect of noise	none
Lorenz 2007	survey of results under Krause–Hegelsmann and Deffuant models	global, although discussion of social network dependence is included		Krause–Hegelsmann and Deffuant	opinion (continuous, multidimensional)	uniform for defiant model, weights in the Krause–wheelsman model	comparison between agent based models and density models (which can be interpreted as limit case for infinitely many agents)	none
Nowak 1990	simulations for the social impact theory of Latane	2D		Nowak–Latane model	opinion (binary); persuasiveness; supportiveness	interaction strength weighted by `social distance'		some remarks on qualitative social opinion phenomena, no direct comparison with simulations
Nowak 1996	general introduction into social modelling; dynamic social impact	2D, some discussion on 1D case		Nowak–Latane model	opinion (binary); persuasiveness; supportiveness; self-supportiveness	variable strength of interaction, decreasing with distance		none
Pluchino 2005	Opinion Changing Rate model introduction	global		OCR model, stressing dynamic aspects over equilibrium	opinion (continuous, unbounded); natural opinion rate change;	variable strength of interaction, depending on opinion difference	focus on differences in tendencies to change opinions	none
Roehner 2005	supplementing computer opinion models with real world data	presentation of `experimental data' with some discussion of the ways of information spread through formal mass media channels as well as through informal channels (rumours, hearsay, gossip) and some macro-constraints		data on consensus formation in various countries for using cell phones while driving				
Sabatelli 2003a	average opinion dependence in Sznajd model with noise	2D		Sznajd model with noise	opinion (binary)	uniform, NN		none
Sabatelli 2003b	role of update mechanism	2D		modified Sznajd model	opinion (binary)	uniform, NN	synchronous updating role	none

		(synchronous vs. async) and memory in Sznajd model				discusses	
Schulze 2004	results for Sznajd model with global and local interactions	2D	modified Sznajd model	opinion (discrete, with Q available options)	uniform, global for comparison of opinions, the local for conversions	Additional analysis of advertising bias for one of opinions	none
Schweitzer 2000	modelling collective opinion formation by means of active Brownian particles	2D, with moving agents	Nowak–Latane model	opinion (binary); social temperature; strength of influence; self-support; global bias	interaction strength given by social distance	agents are not fixed at locations, they move within the 2D geometry, to minimize the pressure on their opinion	none
Slanina 2003	analytical results for the Sznajd model of opinion formation	network with local and global interactions	Sznajd model	opinion (discrete, with Q available options)	uniform	analytical results for certain simplifications	none
Sobkowicz 2003b	comparison of results for Nowak–Latane model in various geometries and with inclusion of strong leaders	comparison of 2D, random network, scale-free network, hierarchical network	modified Kacperski–Holyst model	opinion (binary); strength; leader strength; external bias	interaction strength given by social distance	dependence of behaviours on position of leader in scale free networks	none
Sobkowicz 2003a	effects of leader's strategy on opinion formation	comparison of, scale-free network and short range network	modified Kacperski–Holyst model with introduction of costs of convincing other agents	opinion (binary); strength; leader strength; external bias, costs and available resources, strategy	interaction strength given by social distance and individual strategies in assigning weights to links	strategies define how each agent uses finite resources to convert others to his opinion, leaders have finite resources	none
Sousa 2004b	consensus formation on a triad scale-free network	special case of scale free network	modification of Sznajd model	opinion (binary and discrete spectrum)	uniform	discussion of opinion changes when conviction comes from uniform opinion of 1, 2, 3 neighbours or 3 neighbours forming a triangle	none
Sousa 2004a	bounded confidence model on a still growing scale-free network	scale free network	Deffuant model, continuous	opinion (continuous)	uniform	network grows at the same time as the opinions are adjusted, but no special effects were discovered	none
Stauffer 2001	review of Sznajd model	2D	Sznajd model	opinion (binary)	uniform	generalization to many possible states is used to explain the distribution of votes among candidates in Brazilian local elections	none
Stauffer 2002	persistence of opinion in the Sznajd consensus model	D-dimensional geometric network; D=1,2,3,4	Sznajd model	opinion (binary)	uniform	study of the number of agents that did not change their opinion	none
Stauffer 2003	Simulation of Consensus Model of Deffuant et al on a Barabasi-Albert Network	scale free network	Deffuant model	opinion (continuous);	uniform	behaviour of scale free network found to be similar to random one	none
Stauffer	scaling law for	scale free	discretized	opinion (Q	uniform	multi-layer	none

2004	defiant model on network scale free network		Deffuant model	discrete opinions);		model representing various age levels; advertising effects	
Sznajd 2000	introduction of Sznajd model	1D chain	Sznajd model	opinion (binary)	uniform		none
Tessone 2004	neighbourhood models of minority opinion spreading	1D, 2D	Galam model, global and near neighbourhood interactions	opinion (binary)	uniform within local cells	effects of synchronous and asynchronous updates discussed	none
Weisbuch 2002	comparison of simulation results for continuous and binary opinions and for local and global interactions	global; 2D local interactions	modified Deffuant model;	opinion (multidimensional, continuous, bounded)	uniform		none
Weisbuch 2004	discussion of inuence of possible social networks topologies on the dynamics of Deffuant model	scale free network	Deffuant model	opinion (continuous)	uniform		none
Weisbuch 2005	discussion of the role or extremists in Deffuant model	global; 2D local interactions; scale free	Deffuant model	opinion (continuous)	uniform		none
Wu 2004	dynamical theory of opinion formation in social network	random graph	forced change of opinion on disagreement	opinion (binary)	uniform	simple analytical treatment and simulations; discussion of effects of some agents with fixed opinions	none

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