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Comparing Extremism Propagation Patterns in Continuous Opinion Models

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Abstract

We compare patterns of extremism propagation yielded by 4 continuous opinion models, when the main parameters vary, on different types of networks (total connection, random network, lattice). In two models the individuals take into account the uncertainty of their interlocutor, and they show similar patterns, with a higher probability of double extreme convergence than in the other couple of models (in which the interlocutor's uncertainty is not taken into account). The addition of noise does not change significantly the results, except that it favours the single extreme convergence in some models. The lattice topology of interactions provides results which are significantly different from the ones obtained with a random network of similar connection density. We identify 3 typical behaviours with a single initial extremist, which help to explain the different results. In particular, we observe that the single extreme convergence is favoured by small shortest paths between all pairs of nodes in the network.

Keywords:

Continuous Opinion, Extremism, Convergence Pattern

Introduction

1.1

In recent years, we have proposed several models of opinion dynamics and used them to model the influence of individuals with extreme opinion in a population of moderates ([Deffuant et al. 2002](#), [Amblard and Deffuant 2004](#), [Weisbuch et al. 2002](#), [Weisbuch et al. 2005](#)). These models rely on several general qualitative hypotheses on the agent behaviour:

- Agents have an opinion which varies between some bounds,
- An agent takes into account only opinions which are not too far from his own,
- There are extremists agents, which are very convinced of their opinion (they do not change easily), and their opinion is extreme (at the bounds or very close to the bounds).

1.2

It is implicit in these hypotheses that a distance between opinions can be defined, and that a threshold on this distance determines if the agent will be influenced. The classical binary models in which the opinion is either 0 or 1 (e.g. [Galam and Moscovici 1991](#), [Young 1998](#)), are not appropriate to express the richness of these hypotheses, because in these models the opinion distance can only be 0 or 1. Nevertheless, there is a wide variety of possibilities to express mathematically these qualitative hypotheses.

1.3

The simplest is the bounded confidence model ([Hegselmann & Krause 2002](#), [Deffuant et al. 2001](#); [Weisbuch et al. 2002](#)). In this model, each agent has a threshold in addition to its opinion (sometimes interpreted as an

uncertainty), and it ignores opinions which are at a distance beyond this threshold. In Hegselmann & Krause (2002), an agent takes into account an average influence of all its neighbours, whereas in Deffuant et al. (2001) only pair interactions are simulated. The consequences of this choice are studied in Urbig & Lorenz (2004). More recently, Hegselmann & Krause (2005) studied other variants of averaging within the bounded confidence model. Stauffer et al. (2004) proposed a discrete version of the bounded confidence model and Urbig (2003) a version where agents have a discrete expression of their continuous opinions.

1.4

This model is generalised to a case where agents can have different uncertainties: extremists are initialised with an opinion equal to one of the bounds, and a very low uncertainty, moderates with a randomly chosen opinion and a larger uncertainty. The dynamics on uncertainties can be very similar to the one on opinions, as in Weisbuch et al. (2002).

1.5

It is possible to consider different types of influence dynamics which build on this initial principle. For instance, the Smooth Bounded Confidence model (Deffuant et al. 2004), is another mathematical possibility for expressing the general hypothesis that the influence decreases very rapidly to zero when the distance of opinions is above a threshold. The Relative Agreement (RA) model (Deffuant et al. 2002) expresses the additional hypothesis that an agent takes into account the uncertainty of its interlocutor, and that it will be more influenced if its interlocutor is more certain than itself. Again, other possibilities could have been chosen.

1.6

Edmonds (2006) suggests that the general behaviour of these models is very sensitive to contingent mathematical choices, and to the addition of noise. The objective of this paper is to investigate, on a limited set of chosen models, the sensitivity of the convergence patterns to the modelling choices. However, we consider that the type of noise proposed by Edmonds is in contradiction with the hypotheses of the models. We argue that a noise which is moderated by the uncertainty of the agents respects these hypotheses, and is therefore more appropriate to test the model robustness to noise.

1.7

We limited our investigation to the continuous opinion dynamics models. We chose four of them:

- bounded confidence model (BC) in which we add some dynamics on the uncertainties like in Weisbuch et al. (2002),
- Gaussian bounded confidence (GBC), in which the opinion and uncertainty influence depends on a Gaussian function of the difference of opinions,
- relative agreement (RA) model, in which the influence takes into account the interlocutor uncertainty and the overlap between both segments of opinions,
- a new variant of the Gaussian bounded confidence model in which we multiply the influence by a Gaussian function of the interlocutor uncertainty (GBCU).

1.8

We perform the same systematic experiment designs on these models, in which we vary the essential parameters, for 3 networks of interactions: total connection, random Erdos-Renyi network with 4 neighbours on average, lattice. We then compare the patterns of types of convergence, considering the 3 types that we identified in previous work (moderate convergence, double extreme convergence, single extreme convergence).

1.9

The paper is organised as follows. We first provide the details on the different models and on the indicators we use to define the types of convergence (which are a bit different from the ones used in previous papers). Then, we present the convergence results from systematic experiment designs where the uncertainty of the moderates and the initial proportion of extremists vary, for a given value of μ , the intensity of interactions. We then propose some results about the important influence of μ (which was already pointed out by Urbig and Lorenz (2004)). This leads us to focus on the difference of convergence patterns that appear between the lattice and random network with the same average connectivity. Understanding this difference leads us to investigate the cases of single initial extremist in the population and to relate this to a closer analysis of the network properties.



The considered models of continuous opinion influence

Features which are common to all the model variants

2.1

The considered models share the following aspects :

- The population includes N individuals, each having a continuous opinion and a continuous uncertainty ,
- The population includes a proportion p_e of extremists. Half of the extremists are initialised with opinion -1 , and the other half with opinion $+1$, all with uncertainty u_e ,
- The other agents, called "moderate", are initialised with opinions uniformly distributed between -1 and $+1$, and with uncertainty $U > u_e$,
- A time step is defined by the interactions between N randomly chosen pairs of connected individuals.

2.2

Moreover, for all the models, when an individual with opinion x and uncertainty u meets another individual of opinion x' and uncertainty u' , the modifications of the individuals uncertainties and opinions follow the same type of function. They depend on a kernel function of x , x' and u' , which takes u as a parameter. Let k_u be this function.

$$\begin{aligned} x &:= x + \mu k_u(x, x', u') \cdot (x' - x) \\ x' &:= x' + \mu k_{u'}(x', x, u) \cdot (x - x') \end{aligned} \quad (1)$$

$$\begin{aligned} u &:= u + \mu k_u(x, x', u') \cdot (u' - u) \\ u' &:= u' + \mu k_{u'}(x', x, u) \cdot (u - u') \end{aligned} \quad (2)$$

In the next paragraph, we list the variants of the models corresponding to different choices for k_u .

Variants of the kernel function

Bounded Confidence (BC)

2.3

The initial bounded confidence model including only equations (1), was modified, for instance in Weisbuch et al. (2005), to include equations (2) on the uncertainties. In this case, the kernel function is independent of u' , it is of Heaviside type :

$$\begin{aligned} k_u(x, x', u') &= h_u(x, x') = 1 \quad \text{if } |x - x'| < u \\ k_u(x, x', u') &= h_u(x, x') = 0 \quad \text{otherwise} \end{aligned} \quad (3)$$

The Gaussian Bounded Confidence (GBC)

2.4

In this case, the kernel function has the form of a Gaussian function, and is also independent of u' :

$$k_u(x, x', u') = g_u(x, x') = \exp\left(-\left(\frac{x - x'}{u}\right)^2\right) \quad (4)$$

The relative agreement Model (RA)

2.5

The relative agreement (Deffuant et al. 2002) introduces a new assumption: individuals take into account the uncertainty of their interlocutor, and interlocutors with a low uncertainty (high confidence) tend to be more influential than those with a high uncertainty. The rules use v , the size of the overlap between segments $[x - u, x + u]$ and $[x' - u', x' + u']$.

$$\begin{aligned} v &= \min(x + u, x' + u') - \max(x - u, x' - u') \\ k_u(x, x', u') &= r_u(x, x', u') = \frac{v}{2u'} \quad \text{if } v > 0 \\ k_u(x, x', u') &= r_u(x, x', u') = 0 \quad \text{otherwise} \end{aligned} \quad (5)$$

The value of this function is 1 when the segment $[x' - u', x' + u']$ is totally included in the segment $[x - u, x + u]$. Otherwise, it is lower than 1.

The Gaussian Bounded Confidence Models with influence of uncertainty (GBCU)

2.6

This model expresses the same assumption as the RA model, considering that low uncertainty gives more influence than high uncertainty. This is expressed as follows:

$$k_u(x, x', u') = g_u(x, x') \cdot g_u(u', 0) \quad (5)$$

With this kernel, if u is much smaller than u' , then the change is necessarily small, because then $g_u(u', 0)$ is small.

Introduction of noise

2.7

As suggested in Edmonds (2006), a good way to test the robustness of the results is to check if the addition of some noise changes the results. However, in Edmonds proposal the same noise is applied to all individuals. Therefore, extremists can suddenly strongly change their opinion. This is in contradiction with the assumption of the model stating that the extremists are very convinced of their opinion. To keep the rationale of the model, we consider that the level of random variations in the opinion is moderated by the level of uncertainty.

2.8

To implement this idea, at each time step, with a probability of 0.1, we add to the opinion of each individual, a random number uniformly distributed between -0.3 and +0.3, multiplied by the uncertainty of this individual. In this case, the opinions can be slightly higher than 1 or lower than -1, but this does not introduce any particular problem.



Method for designing experiments and expressing the results

Convergence types

3.1

For all the models, we would like to identify the parameter values leading to the 3 convergence types identified on the RA model: moderate, double extreme and single extreme convergence. These convergence types are illustrated on figure 1. The type of convergence depends on the values of the parameters and we aim to compare the patterns of convergence in the parameter space for the different variants of the model.

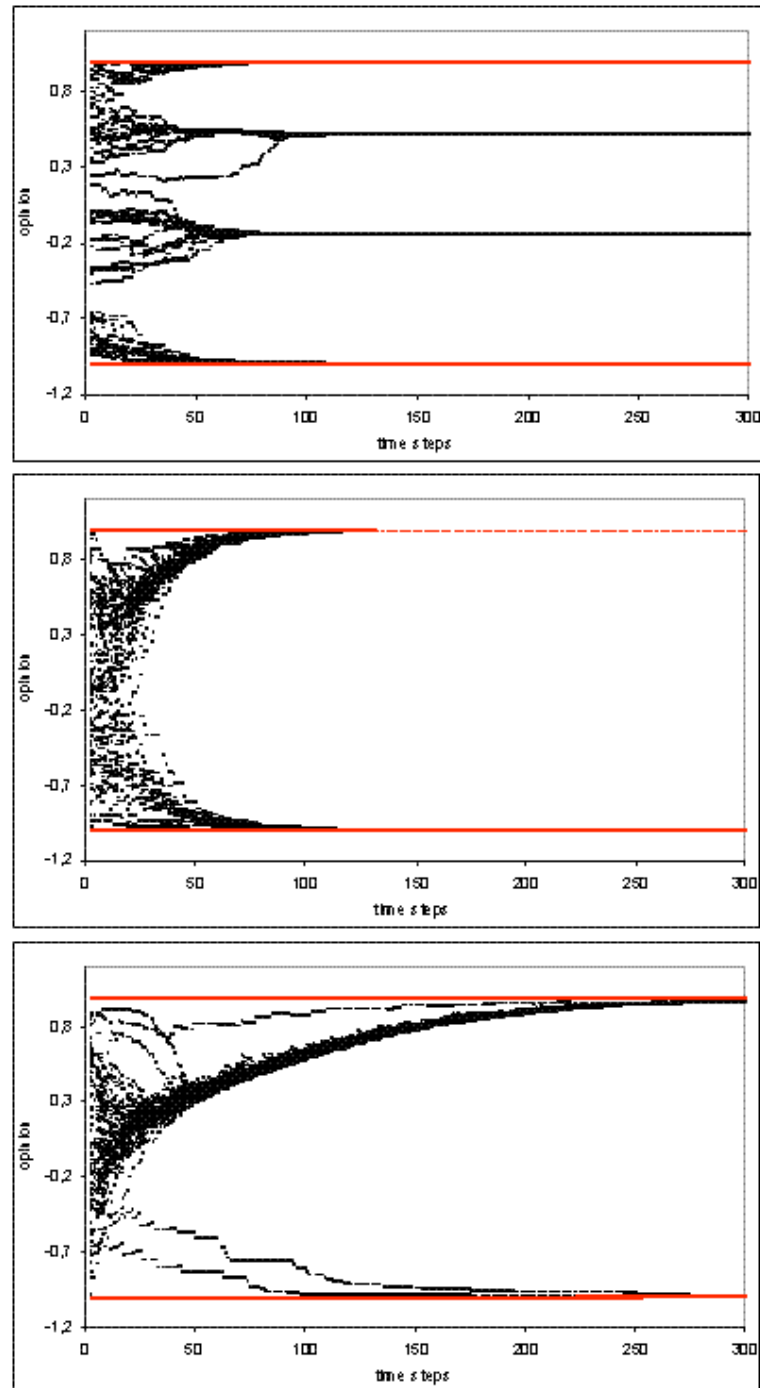


Figure 1. The 3 convergence types. $N = 50$, 2 extremists at $+1$ and -1 , $ue = 0.01$ (represented in red), $\mu = 0.3$. Top: $U = 0.3$, RA model. Moderate convergence ($X=0.62$, $n=3.8$). Middle: $U = 0.9$, RA model. Double extreme convergence ($X=0.99$, $n=1.77$). Bottom : $U = 1.3$, GBCU model. Single extreme convergence ($X=0.97$, $n=1.08$). See paragraph 3.2 for definitions of X and n

Computing the clusters

3.2

We only consider the clusters of initially moderate agents (i.e. initialised with uncertainty U), because our goal is to evaluate the effect on moderates of different configurations of extremists. To compute the clusters, we define a minimum distance ϵ between opinions, below which we consider that they belong to the same cluster. We compute the clusters as groups of agents such that between any couple of agents of opinions x and x' in the group, there is a list of agents in the group of opinions (x_1, x_2, \dots, x_k) making a chain of couples with a distance below ϵ . More precisely:

$$\begin{aligned} |x - x_1| < \epsilon, |x' - x_k| < \epsilon, \\ 1 \leq i \leq k-1, |x_i - x_{i+1}| < \epsilon \end{aligned} \quad (7)$$

In practice, we chose $\epsilon = 0.01$.

3.3

The following pseudo-code can be used to compute the clusters. Initially moderate and extremist agents are tagged at the beginning of the simulation. Method `addAll()` adds all the elements of the argument (supposed to be a list) to the argument (supposed to be a list as well). Method `removeAll()` removes from the recipient all the elements of the argument (both are supposed to be lists).

```
epsilon := 0.01;
todo := indices of initially moderate agents;
clusters := List.new;
while (todo.size > 0):
  currentCluster := List.with(todo[1]);
  newAdded := List.with(todo[1]);
  while(newAdded.size > 0):
    newAdded := List.new;
    for (j := 1, todo.size)
      if (distance(currentCluster, todo[j]) < epsilon) newAdded.add(todo[j]);
    todo.removeAll(newAdded);
    currentCluster.addAll(newAdded);
  end while
  clusters.add(currentCluster);
end while
return(clusters)
```

Characterisation of the convergence type

3.4

To characterise the convergence type of a simulation, we combine two indicators:

- the average of the absolute value of the opinions, noted X , which indicates how extreme is the population,

$$X = \frac{\sum_{i=1}^N |x_i|}{N} \quad (8)$$

the generalised number of clusters, noted n , which is a smooth number of clusters obtained following the method defined in Derrida and Flyvbjerg (1986). Considering a final state of the population involving k clusters of opinion x_i , including a proportion p_i of the initially moderate, the generalised number of clusters is defined by:

$$n = \frac{1}{\sum_{i=1}^k p_i^2} \quad (9)$$

3.5

The generalised number of clusters gives the exact number of clusters when they all include the same proportion of individuals, and intermediate values for intermediate situations. The rationale is that small clusters count less.

3.6

We combine the two indicators to compute the convergence type, with the following rules:

- If $X < 0.8$, then "moderate convergence".
- If $X > 0.8$
 - If $n < 1.25$, then: convergence type = "single extreme".
 - If $1.66 < n$, then: convergence type = "double extreme".
 - If $1.25 < n < 1.66$, then: convergence type = "intermediate between single and double extreme".

3.7

To decide when to stop a simulation, we computed how the value of X is modified after 500 steps. If the modification is higher than 0.01, then we run 500 more steps. Moreover, if $X > 0.9$, we know we can stop the experiment.

Pseudo-code running one experiment

3.8

For sake of clarity, we provide the pseudo-code of one experiment. In this code, `population` is the list of agents, `agent.neighbours` is the list of indices of agent neighbours, `agent.op` is agent opinion, `agent.unc` is agent uncertainty, `kernel()` is function $k_U(x, x', u')$. This function is specific of the model variant.

```

steps := 500;
Xchange := 0.2;
Xlast := computeX();
X := Xlast;
XchangeThreshold := 0.01;
XMax := 0.9;
while ((Xchange > XchangeThreshold) | (X < XMax)):
  repeatNbTimes(steps * population.size):
    agent1 := population[random(1, N)];
    neighbs := agent1.neighbours;
    agent2 := population[neighbs[random(1, neighbs.size)]];
    k12 := kernel(agent1.unc, agent1.op, agent2.op, agent2.unc);
    k21 := kernel(agent2.unc, agent1.op, agent2.op, agent1.unc);
    agent1.setOp(agent1.op + (mu * k12 * (agent2.op - agent1.op)));
    agent1.setUn(agent1.unc + (mu * k12 * (agent2.unc - agent1.unc)));
    agent2.setOp(agent2.op + (mu * k21 * (agent1.op - agent2.op)));
    agent2.set(agent2.unc + (mu * k21 * (agent1.unc - agent2.unc)));
  end repeat;
  X := computeX();
  Xchange := abs(X - Xlast);
  Xlast := X;
end while;

```



Experiments with $\mu = 0.3$

Experimental design

4.1

Following our previous work, we focus on the variation of two parameters: U , the initial uncertainty of the moderates, and p_e , the initial proportion of extremists. We take 51 values of U between 0.2 and 2, and 51 values between 0.01 and 0.21 for p_e . The other parameters are fixed to the following values: $N = 400$ (number of individuals), $\mu = 0.3$ (intensity of changes), $u_e = 0.01$ (uncertainty of the extremists). We preferred to make a refined grid of experiments rather than averaging on several replicas for each set of parameter values, because it gives more directly an idea of the variability of the results.

Bounded Confidence (BC) and Gaussian Bounded Confidence (GBC) models

Total connection

4.2

Figure 2 shows the results for the simple and Gaussian bounded confidence models in total connection, with or without noise. The main features that we notice are:

- The global shape of the parameter zone corresponding to double extreme convergence (blue dots) is similar in both models.
- The convergence to a single extreme (green dots) is much more frequent with the GBC model.

The addition of noise increases the frequency of single extreme convergence in the BC model, and does not change significantly the pattern of the GBC model.

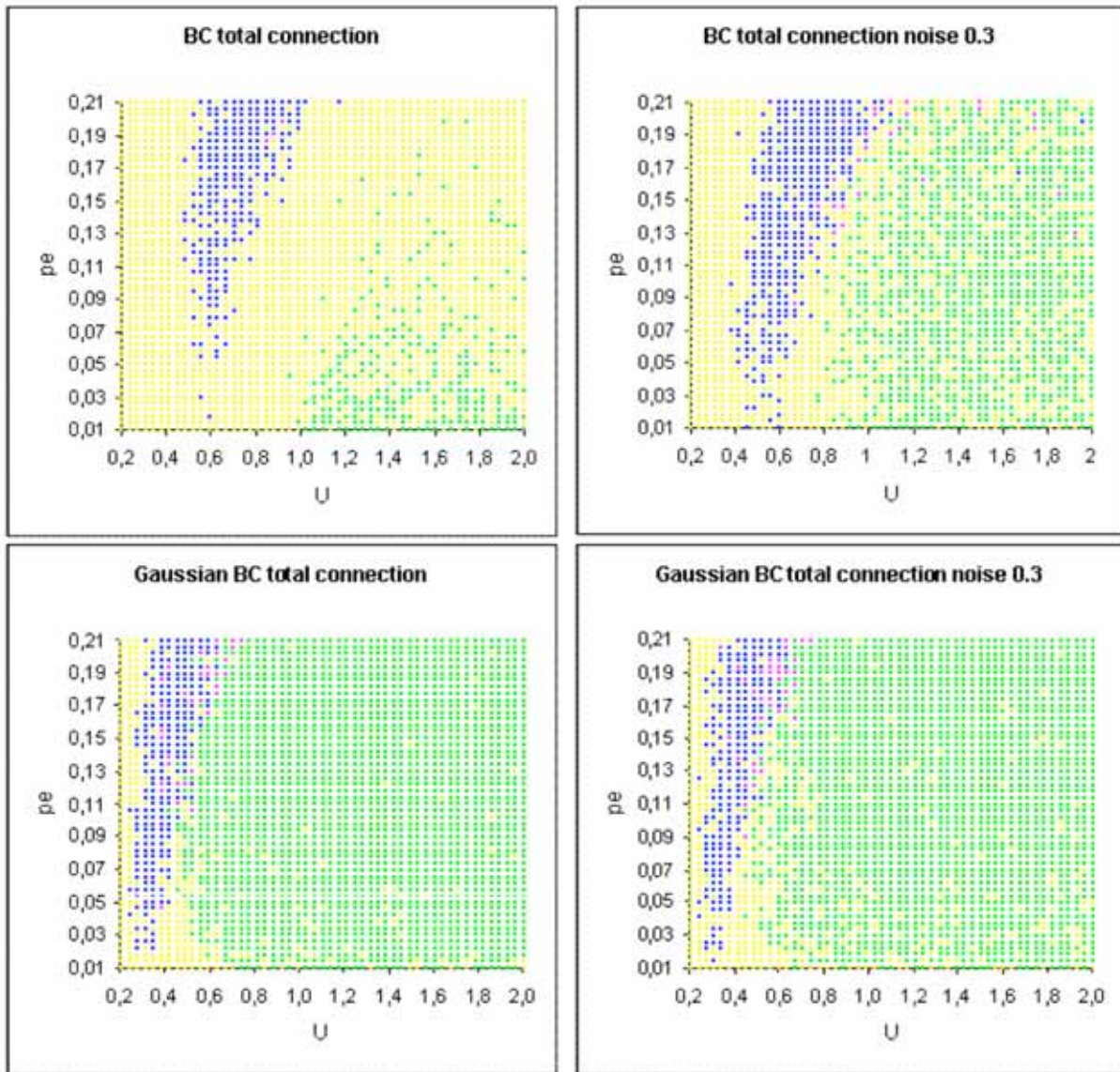


Figure 2. BC and GBC models in total connection. Each dot represents one simulation, and the colour codes for the convergence type: • single extreme, • double extreme, • between single and double extreme, • moderate. Number of individuals: $N = 400$, intensity of interactions: $\mu = 0.3$, uncertainty of extremists: $u_e = 0.01$

Random network with 4 neighbours on average

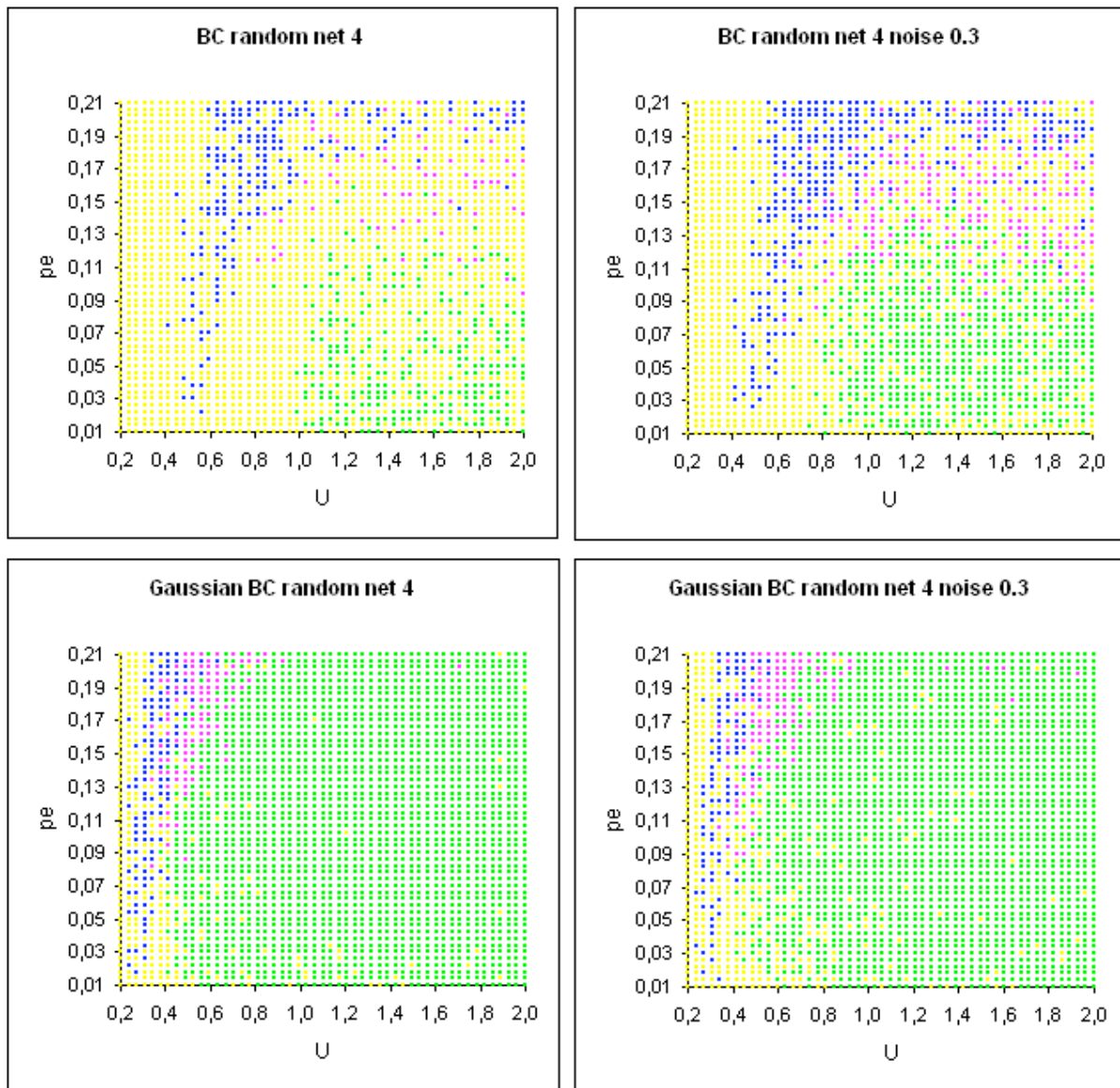


Figure 3. BC and GBC models on random networks with 4 neighbours on average. Each dot represents one simulation, and the colour codes for the convergence type: • single extreme, • double extreme, • between single and double extreme, • moderate. Number of individuals: $N = 400$, intensity of interactions: $\mu = 0.3$, uncertainty of extremists: $u_e = 0.01$.

4.3

The main points to notice in figure 3 are:

- The BC model produces more double extreme convergences, especially for high values of pe , than in total connection,
- Like in total connection, the noise increases the frequency of single extreme convergence of the BC model, and does not significantly affect the GBC model,
- The global patterns of convergence of the GBC model are very similar to the total connection case.

Lattice network

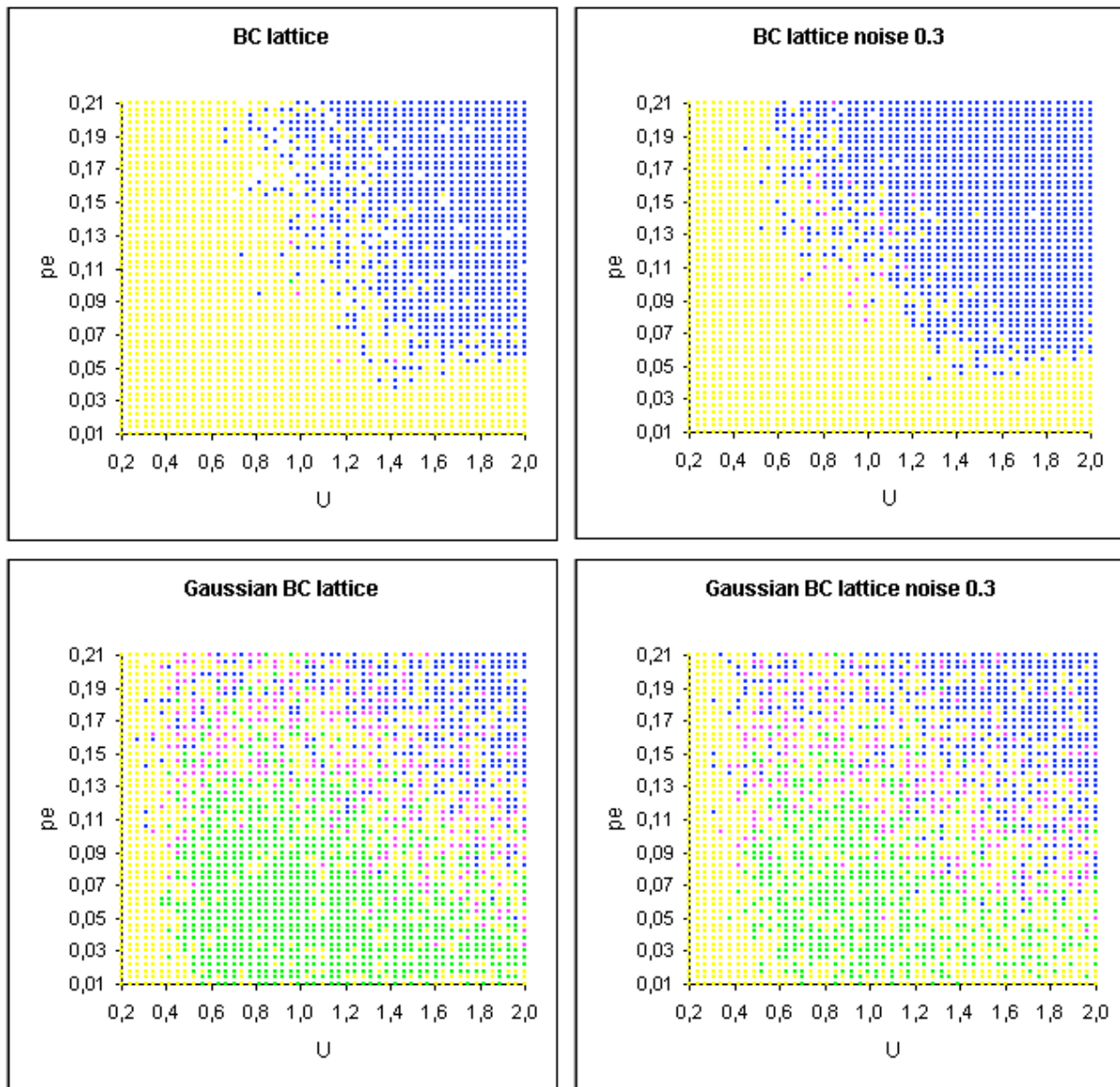


Figure 4. BC and GBC models on a lattice. Each dot represents one simulation, and the colour codes for the convergence type: • single extreme, • double extreme, • between single and double extreme, • moderate. Number of individuals: $N = 400$, intensity of interactions: $\mu = 0.3$, uncertainty of extremists: $u_e = 0.01$

4.4

The main noticeable points in figure 4 are:

- The patterns of convergence on a lattice are very different from the patterns in the totally connected and random network cases.
- For the BC model, the single extreme convergence (green) is almost not found in the lattice case. Only a few intermediate convergences (purple) take place from time to time for U around 1 and a medium proportion of extremists.
- The pattern of double extreme convergence exhibited by the GBC model is similar to the one of the BC model, however, the GBC model produces single extreme convergence in a large part of the parameter space, where the BC model produces a moderate convergence. We also note that the transition between single and double extreme seems to be smooth: a lot of intermediate convergences are located at the boundary between the two zones.
- The noise has no significant influence, except that the GBC seems to produce a bit less frequently single extreme convergences when there is some noise.

Relative Agreement model and Gaussian BC with influence of uncertainty

Total connection

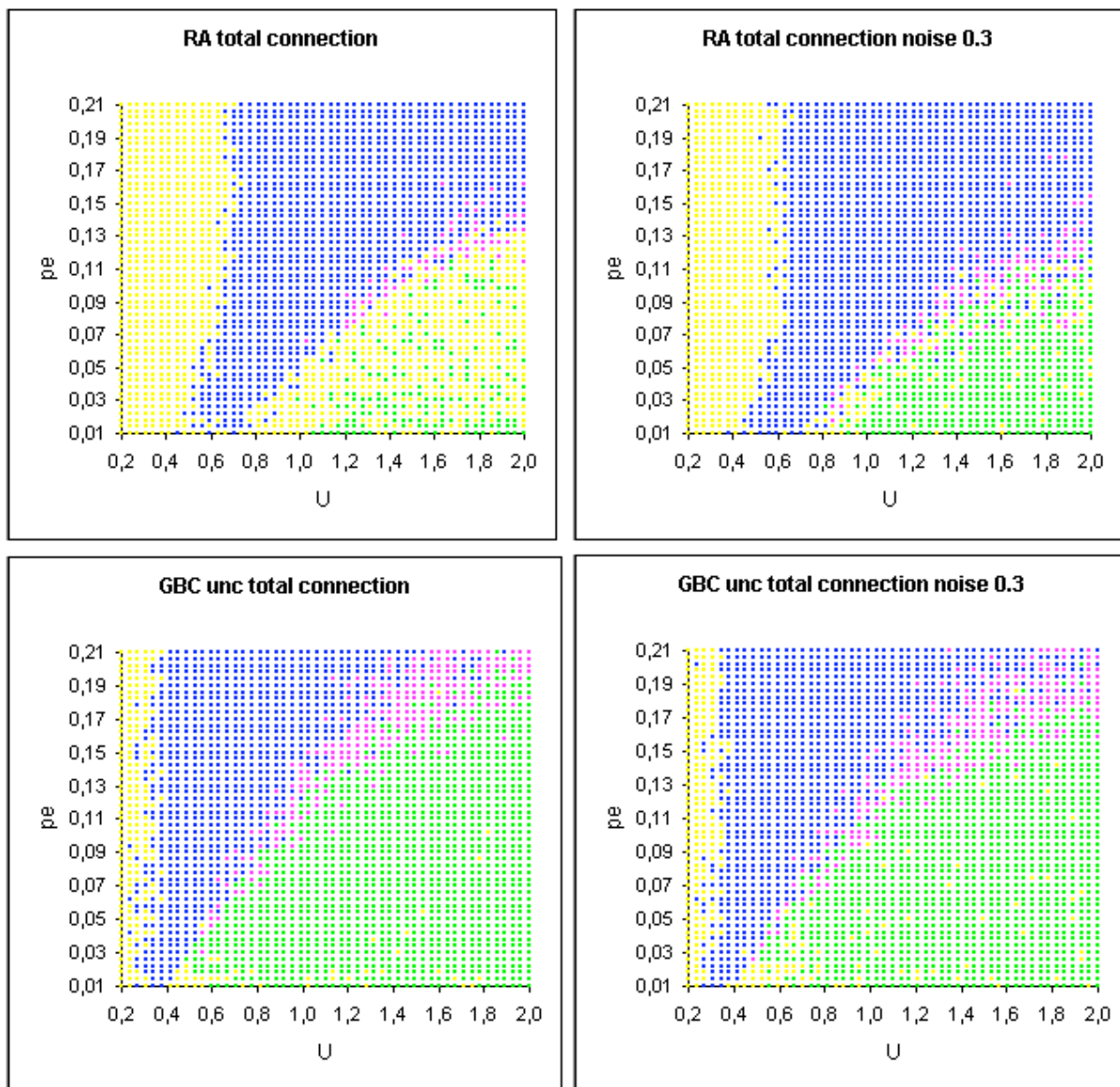


Figure 5. RA and GBCU models in total connection. Each dot represents one simulation, and the colour codes for the convergence type: • single extreme, • double extreme, • between single and double extreme, • moderate. Number of individuals: $N = 400$, intensity of interactions: $\mu = 0.3$, uncertainty of extremists: $u_e = 0.01$

4.5

The main features of figure 5 are:

- The patterns of double extreme convergence are similar for RA and GBCU models,
- The global patterns are significantly different from the ones of the BC and GBC models. In particular the RA and GBCU models produce double extreme convergence for high p_e and high U .
- The RA model produces more frequently the single extreme convergence when there is some noise, whereas the convergence pattern of the GBCU model is almost unchanged by the noise.

Random network with 4 neighbours on average

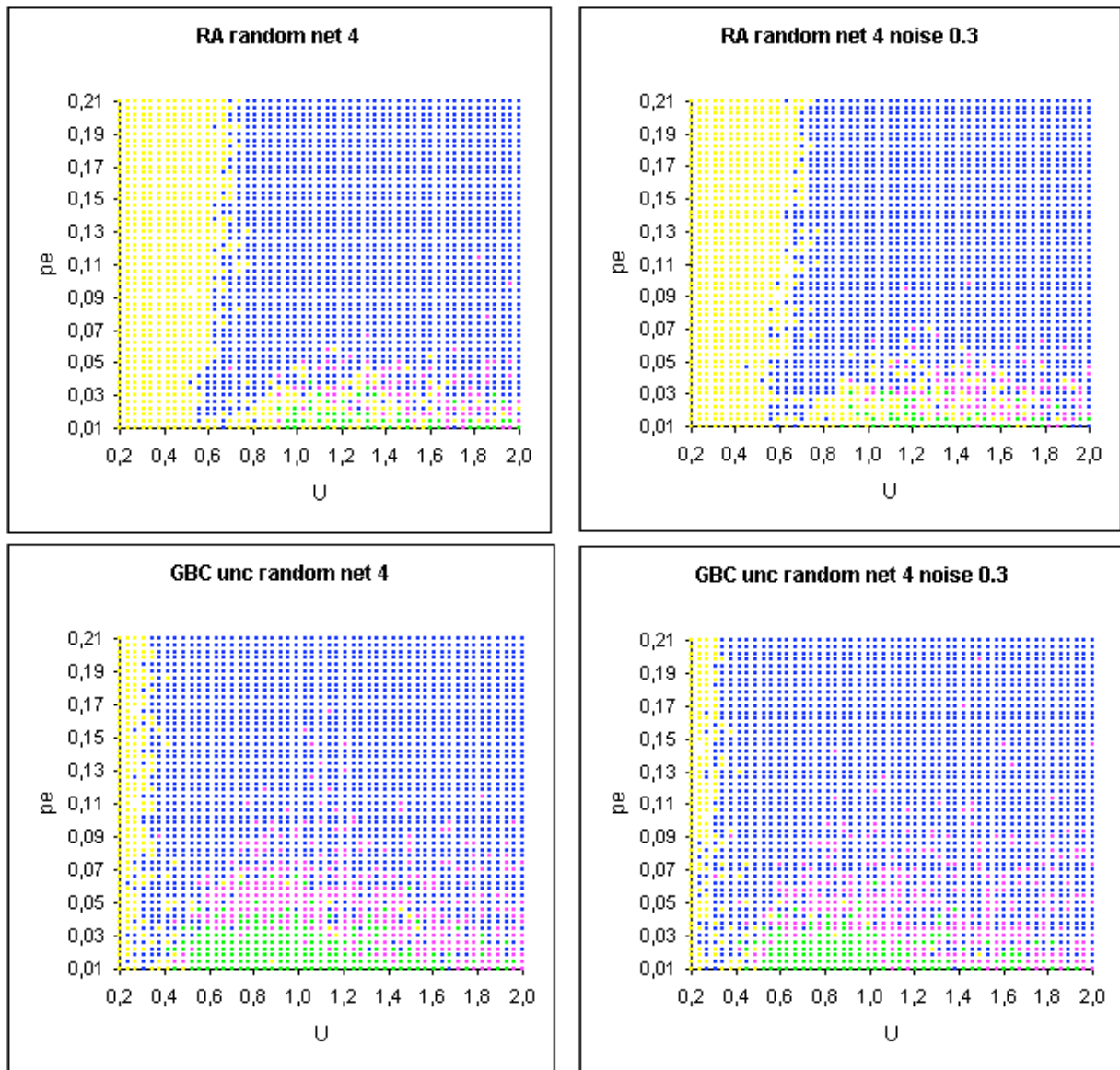


Figure 6. RA and GBCU models on a random network with 4 neighbours on average. Each dot represents one simulation, and the colour codes for the convergence type: • single extreme, • double extreme, • between single and double extreme, • moderate. Number of individuals: $N = 400$, intensity of interactions: $\mu = 0.3$, uncertainty of extremists: $u_e = 0.01$

4.6

The main features of figure 6 are:

- The difference with the totally connected case is bigger than the one observed between the BC and GBC in total connection and random network.
- The global patterns are similar for both models: majority of double extreme convergence, with a small zone of single extreme convergence for low values of p_e .
- The noise does have a significant impact on the results.

Lattice network

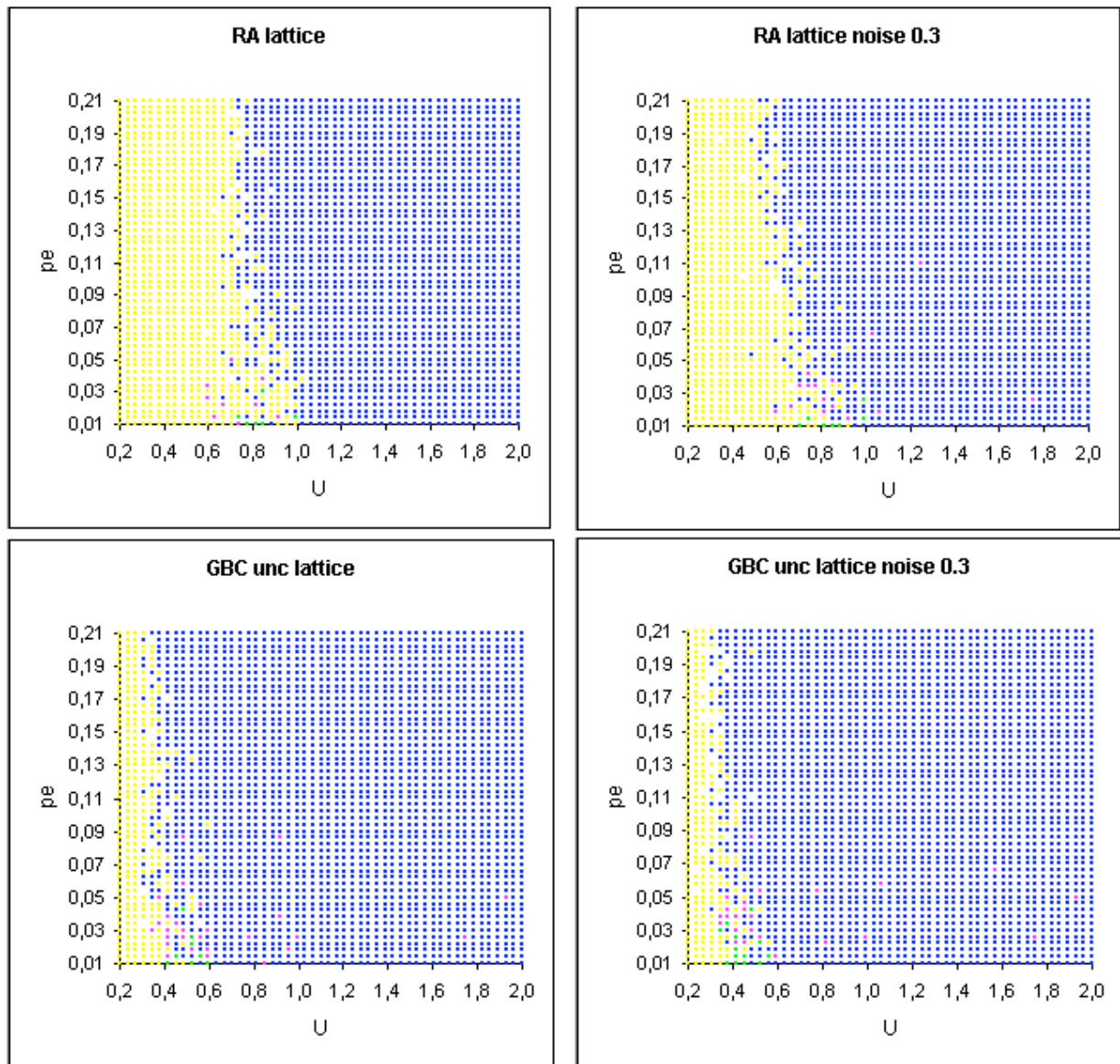


Figure 7. RA and GBU on a lattice. Each dot represents one simulation, and the colour codes for the convergence type: • single extreme, • double extreme, • between single and double extreme, • moderate. Number of individuals: $N = 400$, intensity of interactions: $\mu = 0.3$, uncertainty of extremists: $u_e = 0.01$

4.7

The main features of figure 7 are:

- The global patterns are similar: a large majority of double extreme convergence, with a very small region of single extreme convergence, for U between 0.6 and 1 for the RA model, between 0.4 and 0.6 in the GBCU model, and small p_e . This corresponds to the type of convergence identified in Weisbuch et al. (2005), where none of the extremists of one side manages to diffuse, whereas some of the other side do.
- The noise has no significant impact on the patterns.



Role of parameter μ

5.1

The role of parameter μ was investigated in (Urbig and Lorenz, 2004) on their model generalizing from different versions of BC without extremists. They found that its value can modify the number of final clusters. The role of this parameter on the models with extremists has not been much investigated, and it turns out that it can modify the convergence patterns very significantly.

Results for BC and GBC for high value of μ

5.2

Figures 8 provide the results of the experiments for $\mu = 0.7$, in total connection.

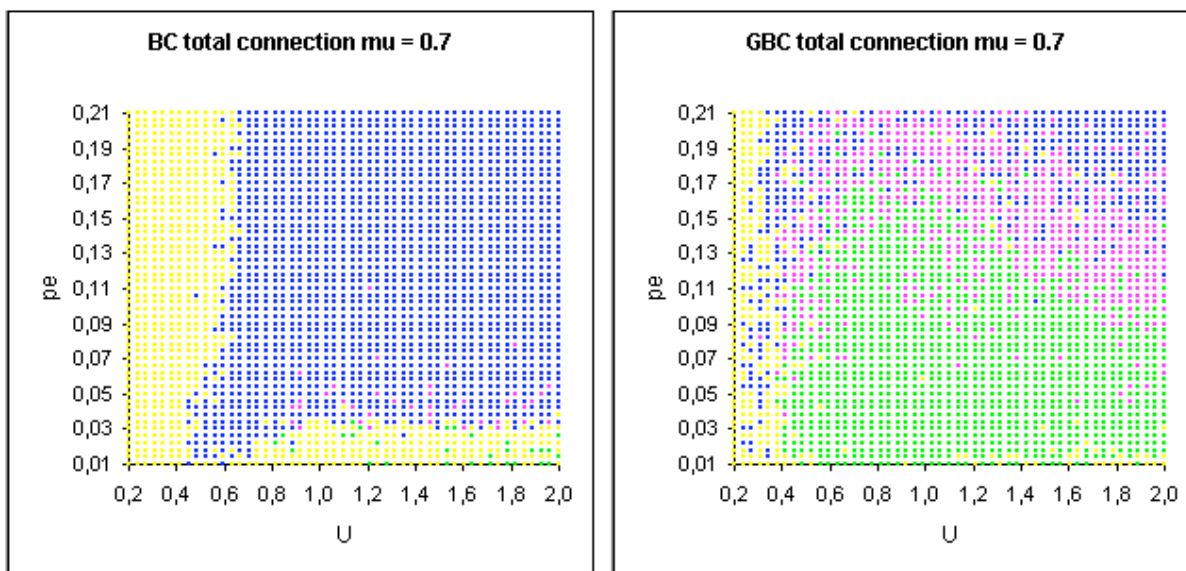
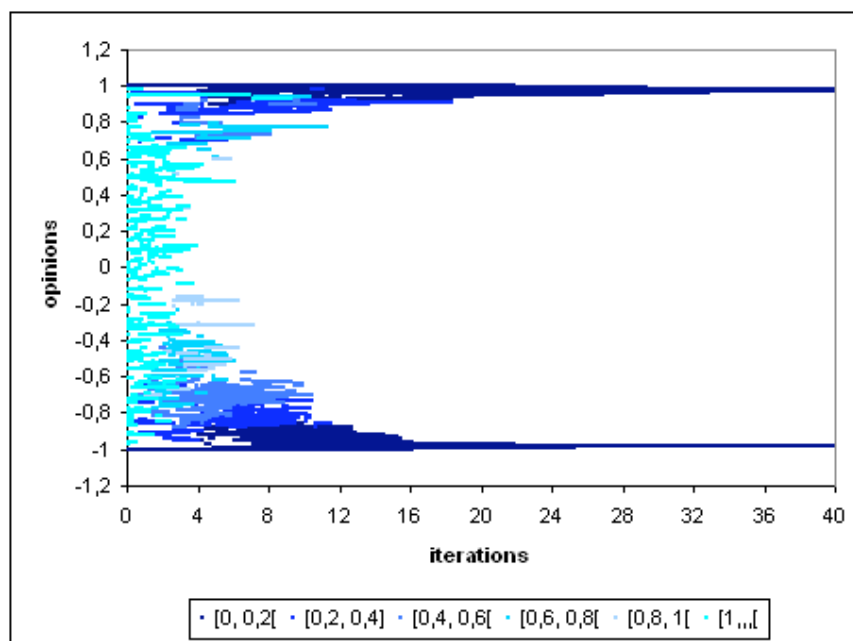


Figure 8. BC and GBC for total connection. Same colour codes as in the previous figures. Number of individuals: $N = 400$, intensity of interactions: $\mu = 0.7$, uncertainty of extremists: $u_e = 0.01$

5.3

The picture of convergence changes dramatically, compared to the one obtained with $\mu = 0.3$. For the BC model, the double extreme convergence always takes place when $U > 0.6$ and $p_e > 0.04$. The picture becomes therefore more similar to the one obtained with RA and GBCU models. The GBC model reacts differently. Although the double extreme convergence takes place in a larger part of the parameter space (especially when p_e is large), the change is not as big as for the BC model.



E

Figure 9. Example of double extreme convergence for the RA model in total connection, with intensity of interactions: $\mu = 0.7$. $N = 100$, uncertainty of extremists: $u_e = 0.01$, $p_e = 0.15$, $U = 1.2$. The colours indicate the value of uncertainties (see the legend). In one interaction, an extremist modifies dramatically the opinion and the uncertainty of a moderate

5.4

As shown on figure 9, the reason the high frequency of double extreme convergence with the RA model is due to the strong change occurring at each interaction with an extremist, which transforms a moderate into an extremist in a few interactions. For the same value of μ , the interactions with the GBCU model are smoother, and the double extreme convergence is thus less frequent.

Results for RA and GBCU with small value of μ

5.5

As shown on figure 10, the boundary of the double extreme parameter zone seems to be linear when the value of μ gets small (0.02).

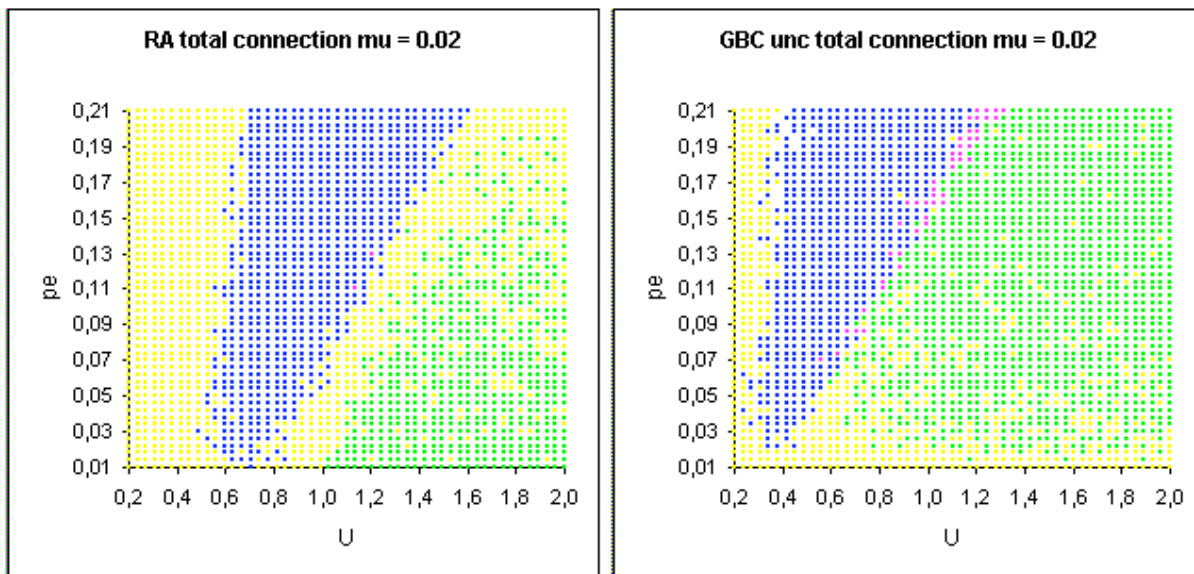


Figure 10. RA and GBCU for total connection. Same colour codes as in the previous figures. Number of individuals: $N = 400$, intensity of interactions: $\mu = 0.02$, uncertainty of extremists: $u_e = 0.01$

5.6

We note that this boundary is significantly different from the one obtained with BC and GBC, even for $\mu = 0.3$. With this small value of μ , the double extreme convergence is obtained for high U when, at the beginning of the simulation, the average decrease of the moderates uncertainty is fast enough compared to the tendency to form a single narrow cluster. In this case, when the average U becomes lower than 1, the moderates are still dispersed, half the population is attracted by one extreme, and the other half by the other extreme (see figure 11).

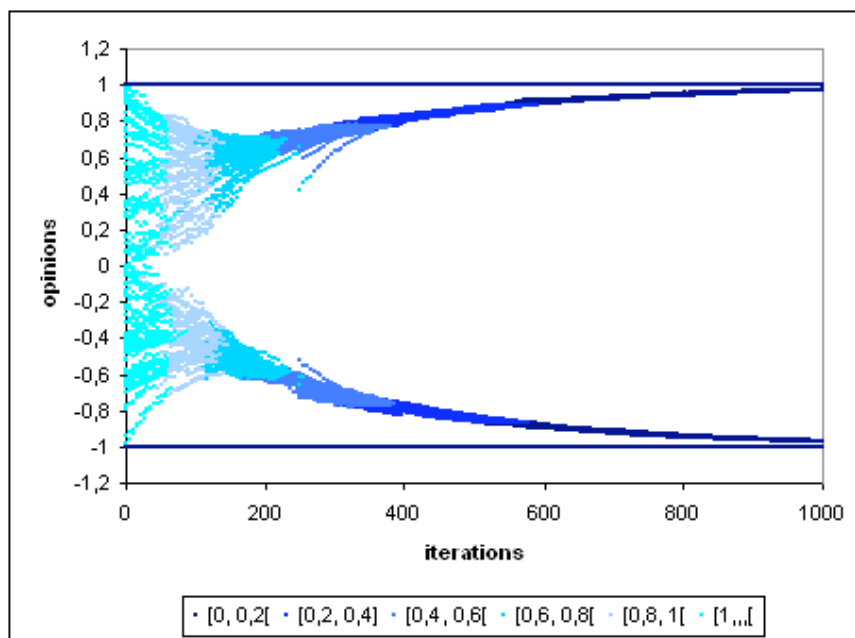


Figure 11. Example of double extreme convergence for the RA model in total connection, with intensity of interactions: $\mu = 0.02$. $N = 100$, uncertainty of extremists: $u_e = 0.01$, $p_e = 0.15$, $U = 1.2$. The colours indicate the value of uncertainties (see the legend). The split occurs when the average uncertainty is below 1

5.7

This does not take place with the BC or GBC models, because the clustering effect is too strong compared to the uncertainty decrease: when the average U becomes lower than 1, the moderate opinion form a narrow cluster close to 0, which cannot split.

Larger population

6.1

The effect of the population size on the convergence patterns has not been much studied yet. Therefore, we made some experiments on populations with 2500 agents. Figure 12 shows the cases in which we found significant differences compared with a population of 400 agents: For the BC and RA model in total connection, and the GBC model on a lattice network, the frequency of single extreme convergence is significantly lower for a population of 2500 individuals.

6.2

To understand the differences observed on the RA and BC models in total connection, one must consider the process of single extreme convergence. As shown in Deffuant et al. (2002), this process involves first a central clustering of the moderate opinions, and then some agents are regularly attracted to the extremes, inducing fluctuations of this central clustering and a general decrease of the uncertainty. When the cluster involves a larger number of agents, individual fluctuations have a lower effect on the cluster.

6.3

The GBC model does not show a different final pattern of convergence in total connection, with the larger population. However, the convergence is much slower (5000 to 10000 time steps) than with a population of size 400 (1000 to 2000 time steps), which is coherent with our explanation.

6.4

To understand the difference of convergence pattern observed on the GBC model on a lattice, we need to focus more on the particular case of lattice networks.

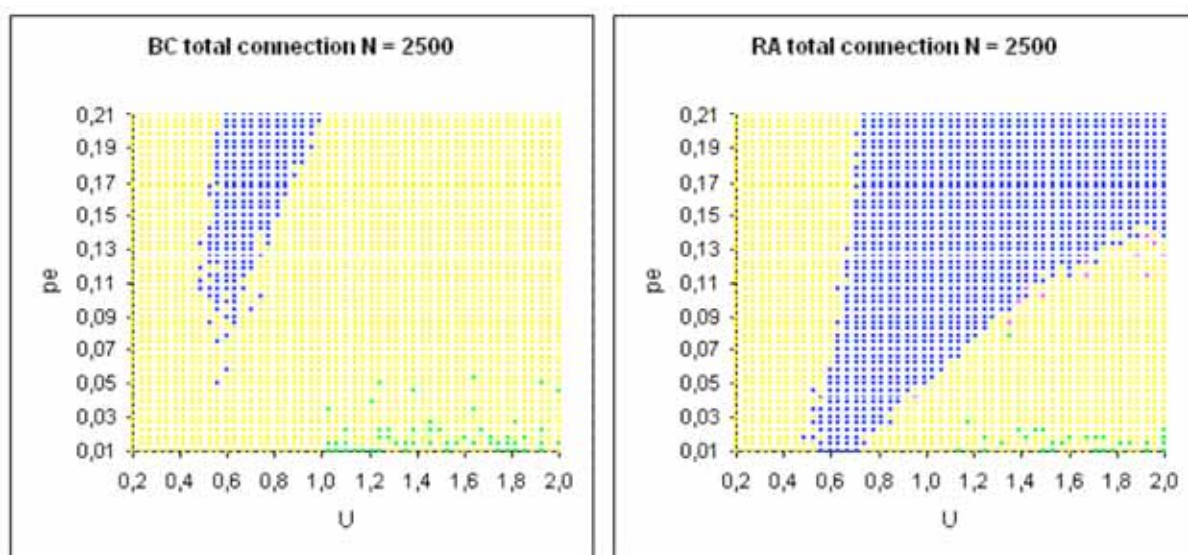


Figure 12. Convergence patterns with a population of size 2500 for RA and BC models in total connection ($\mu = 0.3$). In both cases, the frequency of single extreme convergence is smaller than with a population of size 400

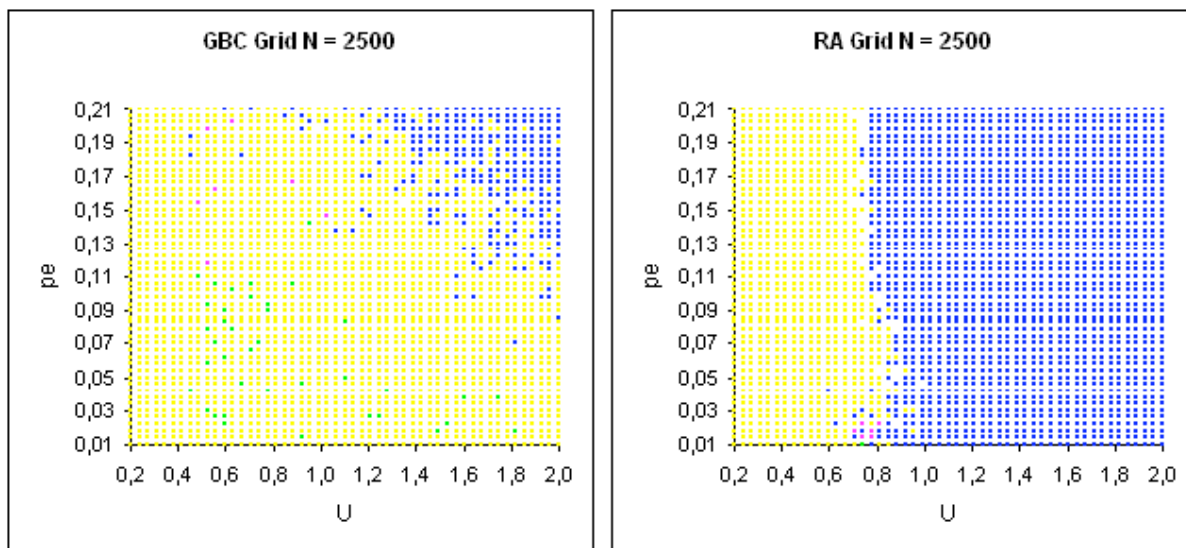


Figure 13. Convergence patterns with a population of size 2500 for RA and GBC on a lattice network ($\mu=0.3$). The RA model does not show significant difference with the case of a population of size 400, whereas the GBC model shows much less single extreme convergence

Explaining the difference between lattice and random network

Difference between lattice and random network (with average neighbourhood of size 4)

7.1

The difference of behaviour between the lattice and the random network with the same size of neighbourhood is particularly striking in the previous experiments. For instance, we note that the single extreme convergence takes place in a very limited zone of parameters for the GBCU and RA models on a lattice, whereas it is much larger on the random network. The difference holds for small values of μ , as shown on figure 14 and 15.

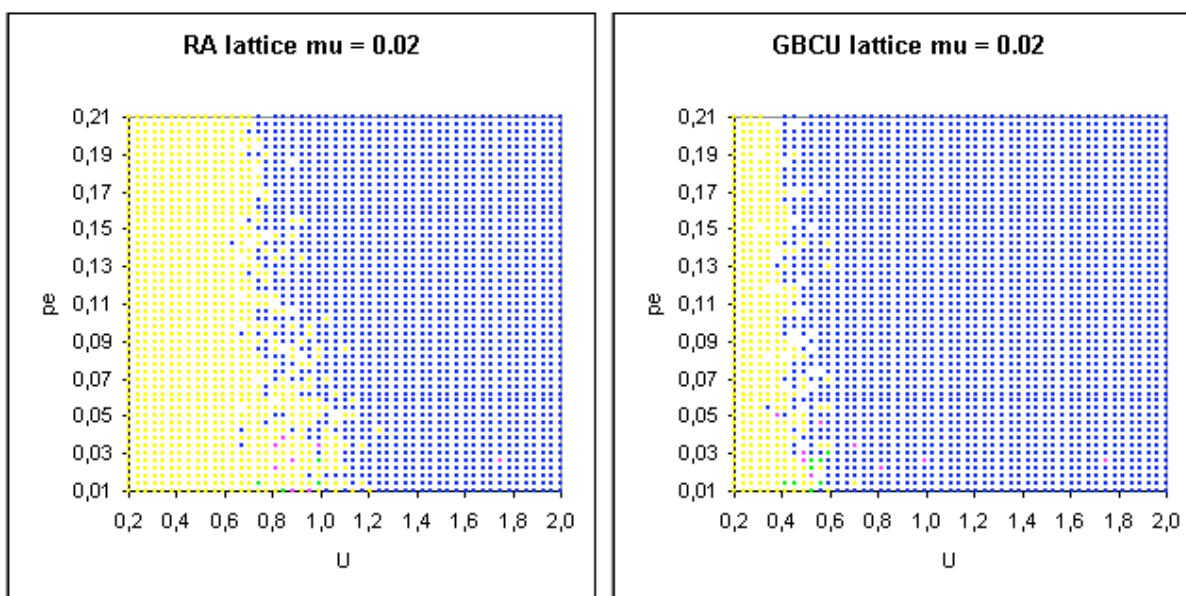


Figure 14. RA and GBCU for a random network with 4 neighbours on average. Same colour codes as in the previous figures. Number of individuals: $N = 400$, intensity of interactions: $\mu = 0.02$, uncertainty of extremists: $u_e = 0.01$

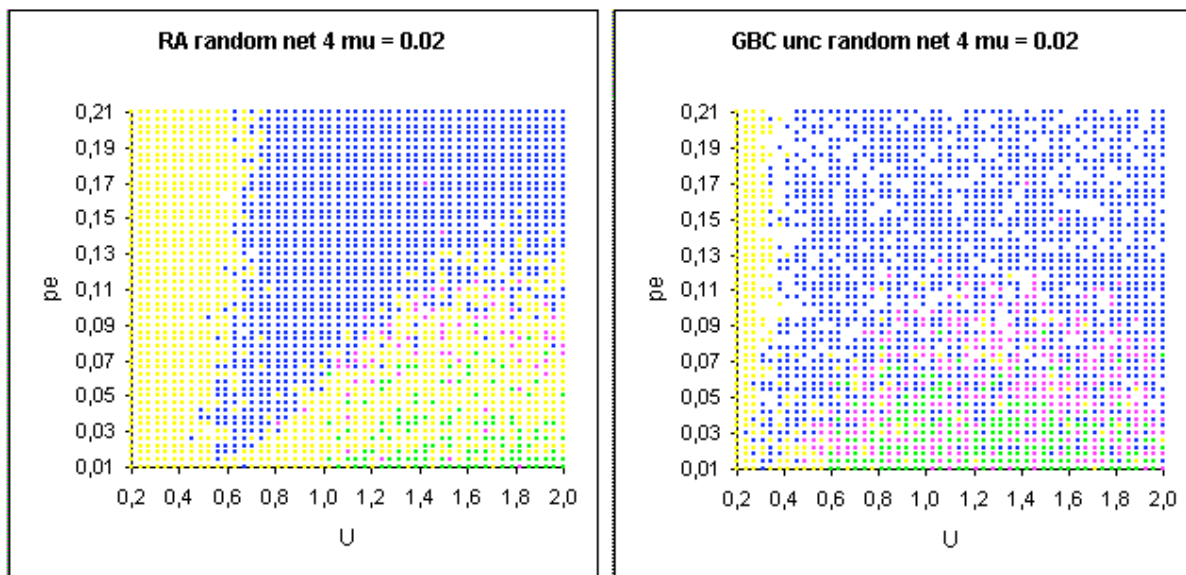


Figure 15. RA and GBCU for a random network with 4 neighbours on average. Same colour codes as in the previous figures. Number of individuals: $N = 400$, intensity of interactions: $\mu = 0.02$, uncertainty of extremists: $u_e = 0.01$

7.2

In order to better understand these differences, we consider the case where there is only one extremist at the beginning of the simulation (we build on an approach that we already adopted in [Weisbuch et al. 2005](#)).

Typical behaviours on a lattice with only one extremist

Three typical behaviours appear: Strong local diffusion (figure 16), weak local diffusion (figure 17) and global diffusion (figure 18).

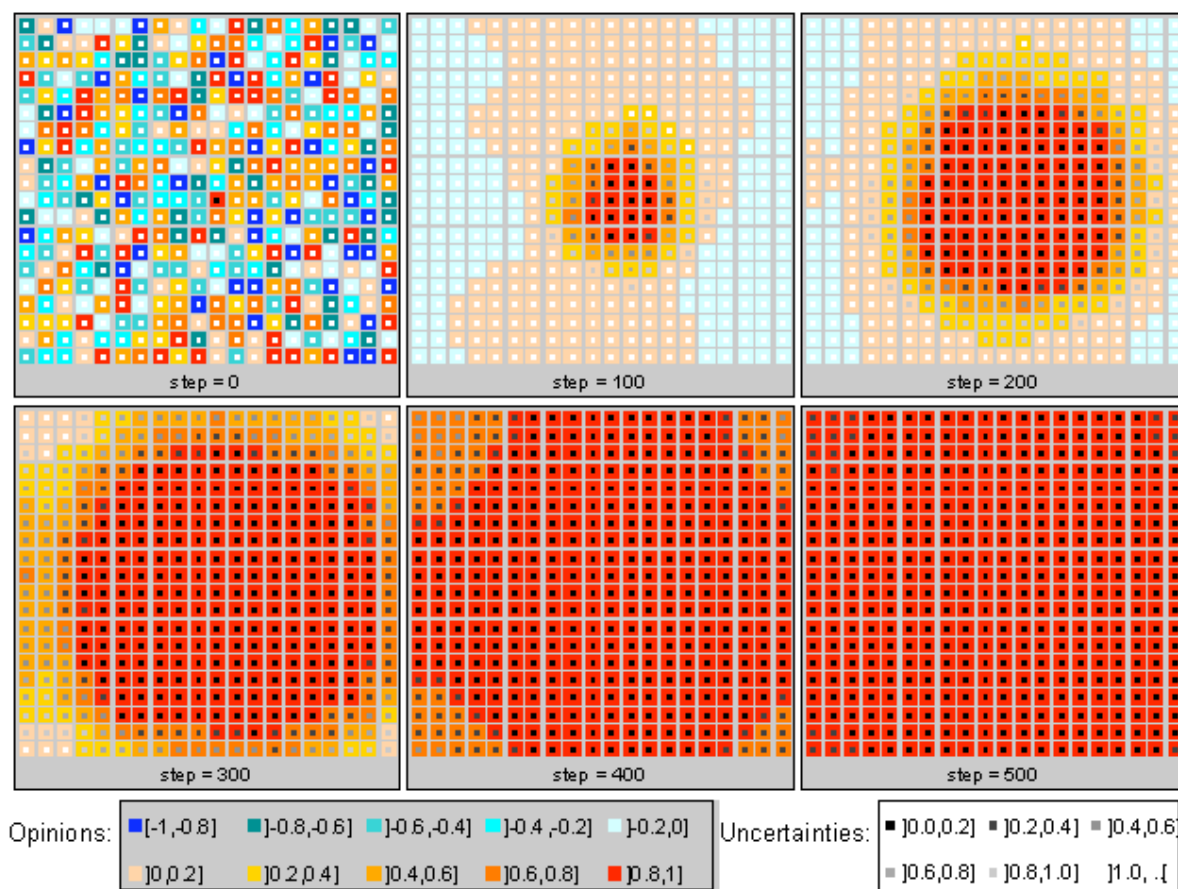


Figure 16. Strong local diffusion. Example of simulation for the RA model on a lattice with only one extremist at the beginning. $\mu = 0.1$, $u_e = 0.01$, $N = 400$, $U = 1.5$. The colours represent the opinion level (between -1 and +1), and the grey levels the uncertainty. Each individual is connected to four neighbours.

Rapidly the extremist propagates its extreme opinion and certainty to its neighbours, which do the same.
 The final mean absolute value of the opinion is $X = 0.95$

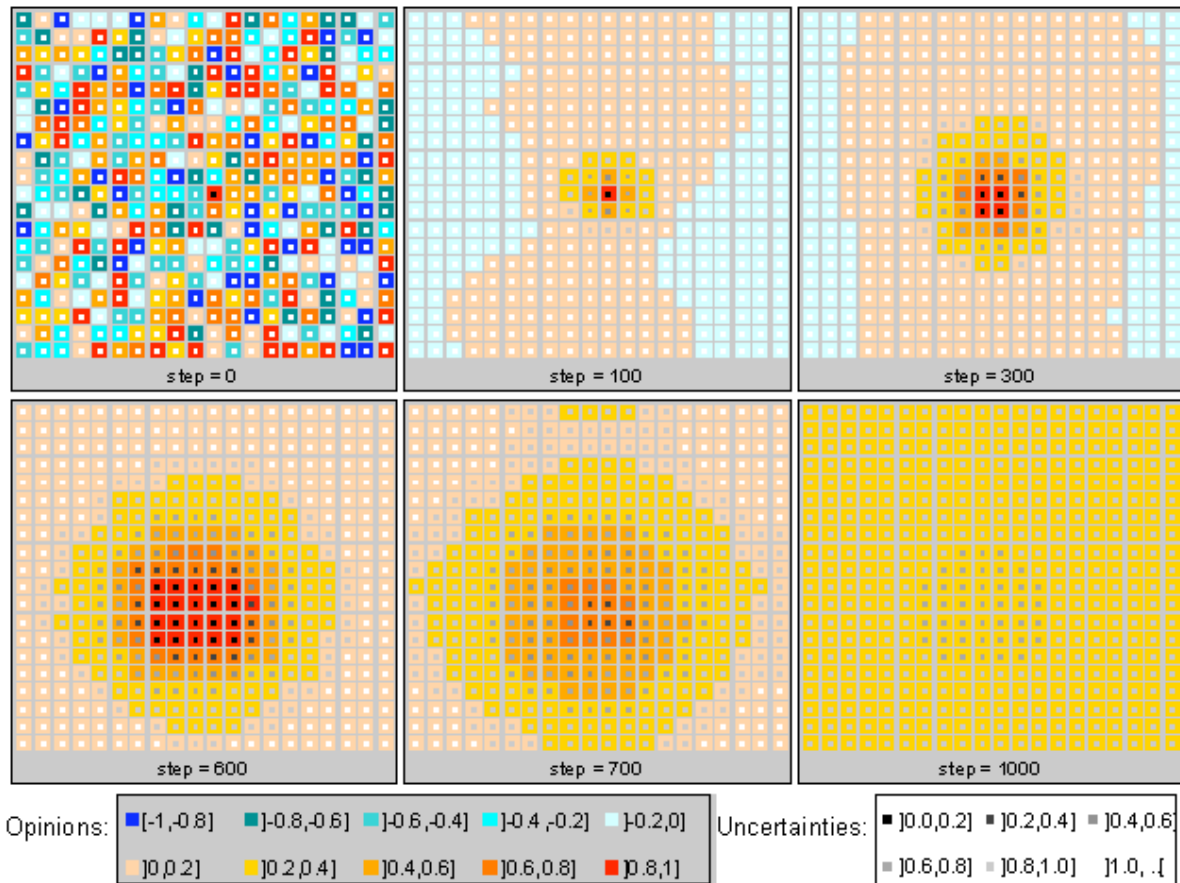


Figure 17. Weak local diffusion. Example of simulation for the BC model on a lattice with only one extremist at the beginning. $\mu = 0.1$, $u_e = 0.01$, $N = 400$, $U = 1.5$. The colours represent the opinion level (between -1 and +1) of the individuals located on a lattice. Each individual is connected to four neighbours. Rapidly the neighbours of the extremist have an opinion which is close to the extreme, and they influence the extremist, increasing its uncertainty. The extremist then becomes unstable, and is attracted by the moderates. The final mean absolute value of the opinion is $X = 0.27$

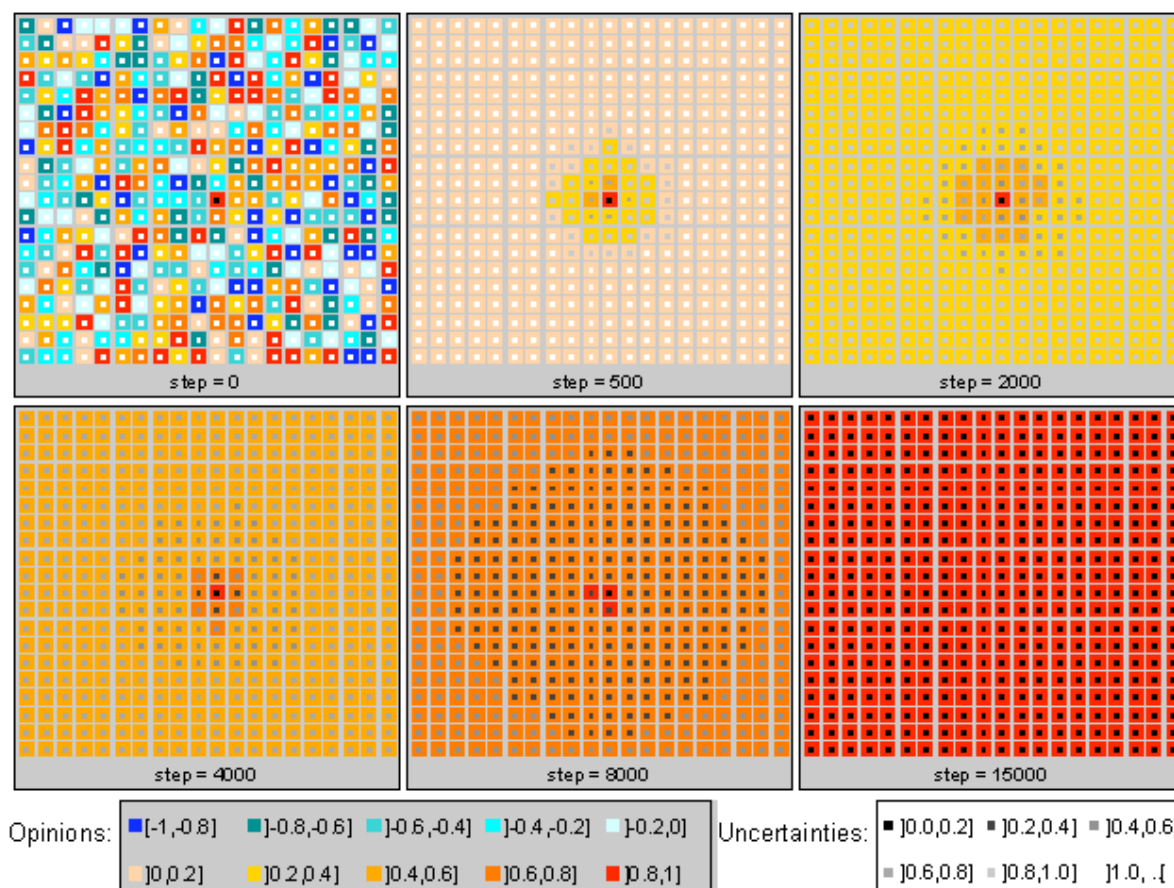


Figure 18. Global diffusion. Example of simulation for the GBC model on a lattice with only one extremist at the beginning. $\mu = 0.1$, $u_e = 0.01$, $N = 400$, $U = 1.5$. The colours represent the opinion level (between -1 and +1) of the individuals located on a lattice. Each individual is connected to four neighbours. The moderates tend to keep a similar opinion, which is globally and very slowly influenced by the extremist. The final mean absolute value of the opinion is $X = 0.91$. The role of μ is crucial here, because for $\mu = 0.3$ we get a weak local diffusion

7.3

These types of diffusion from one extremist explain the different types of convergence we can get when there are several extremists of both sides at the beginning:

- The strong local diffusion leads to double extreme convergence, because the regions around the extremists of each side are rapidly converted to their extreme opinion. Finally, each individual becomes extremist from one side or from the other, and they are in an almost equal quantity. This type of diffusion is obtained with RA and GBCU models, whatever the value of μ , because in these models the extremists are less sensitive to the influence of the moderates.
- The weak local diffusion generally leads to moderate convergence when the number of initial extremists is small, because the local diffusions initiated by the extremists are not strong enough to be maintained, and they finally become extinct. When the number of extremists gets higher, it can lead to double extreme convergence, because the diffusion can be strong enough to reach the diffusion region of an other extremist. The weak local diffusion takes place particularly in the BC model, when the value of μ is medium or high.
- The global diffusion generally leads a single extreme convergence, because all the moderates keep a similar opinion, which can be captured at a given moment by one of the extremes. This takes place with the GBC model, when the number of initial extremists is not too high, and the population not too large. This explains why the GBC model shows a high number of single extreme convergence for a population of size 400.

7.4

Considering a single initial extremist confirms the very different behaviours of the models on lattice and random networks. For instance, as shown on figure 19, the BC model yields a global diffusion on a random network with 4 neighbours, whereas it yields a weak local diffusion on a lattice.

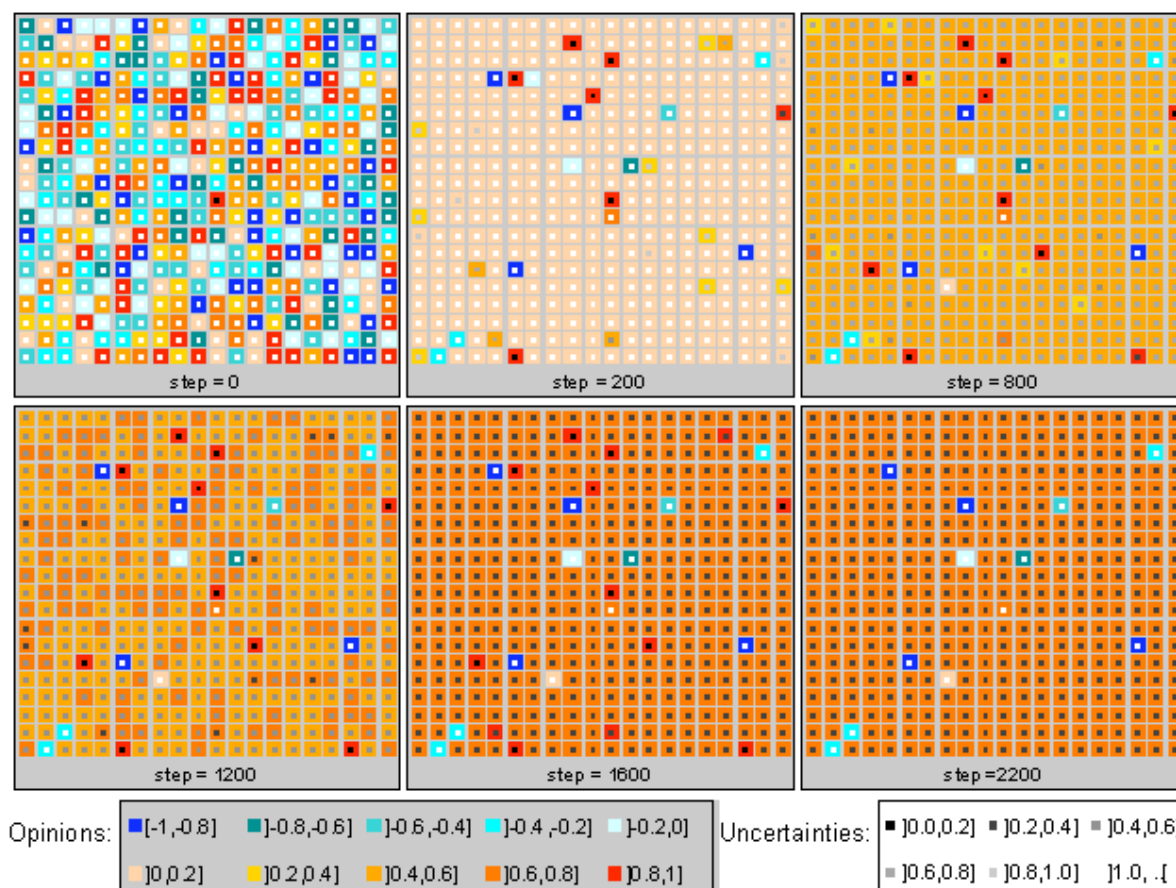


Figure 19. BC model with single initial extremist, random network 4 neighbours, $N = 400$, $\mu = 0.1$, $u_e = 0.01$, $U = 1.5$. The evolution is of the global diffusion type, whereas it is a weak local diffusion on a lattice for the same parameters

Explaining the difference with a closer analysis of the network

7.5

Clearly, the average number of connections is not sufficient to predict the dynamical behaviour of the models, as already observed in (Amblard & Deffuant 2005), and other descriptors should be considered.

7.6

An interesting indicator is the shortest path. As shown on figure 20, the shortest path between two randomly chosen individuals is much shorter in a random network with 4 neighbours (average size: 4.25) than in a lattice (average size: 10.0). This means that all the moderates are much closer to each other in a random network, which favours their clustering, and therefore the global diffusion. This explains why we get a global diffusion with a population with only one initial extremist and a random network with 4 neighbours on average with the BC model (figure 19) whereas it yields a weak local diffusion in the same conditions on a lattice (figure 17). The reason is that the moderates have opinions which remain close to each other, because of their close relations. Therefore, the group of extremists tends to remain separated from the moderates, which prevents it to be influenced by the moderates. We get therefore a global diffusion.

7.7

The GBC model provides a global diffusion even with long shortest paths, because the influence of extremists is smoother, which favours a stronger clustering of the moderates. This explains its high number of single extreme convergences, even on a lattice network. This number decreases when the population size increases, which is coherent with our explanation, because when the population size increases, the shortest paths increase.

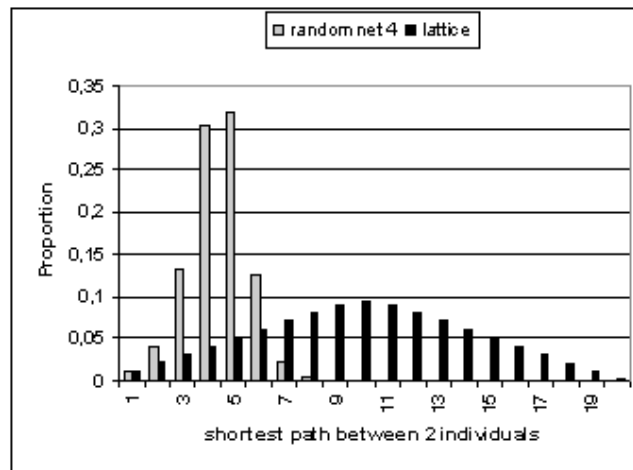


Figure 20. Shortest path distribution. Population size $N = 400$. Average result for 400 trials in the case of the random network

7.8

In order to check the robustness of the shortest path indicator, we performed some experiments on scale-free networks (Barabasi & Reka 1999), which are known to have very small shortest paths. As shown on figure 21, considering a scale-free network with 4 links per node on average produces convergence patterns in which there are less single extreme convergence than on random networks of same link density (average shortest path: 3.6).

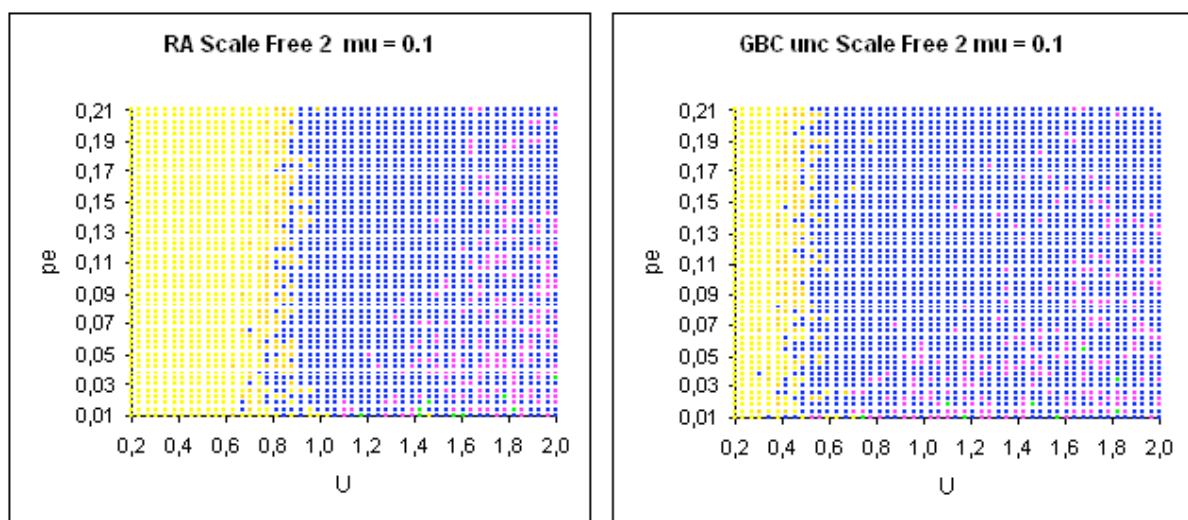


Figure 21. Convergence patterns for RA and GBCU models on scale-free networks with 4 links per node on average (network parameter equals 2). Population size $N = 400$, $\mu = 0.1$

7.9

These results can be related to the properties of scale-free networks. In this scale-free network, a majority of agents have only 2 links and a limited number a much higher number. Moderate agents with only two links are very easy preys if one of their neighbours is an extremist, because the influence of the extremist is not counterbalanced. Therefore, this leads to a tendency to local diffusion, which favours the double extreme convergence.

7.10

These experiments suggest that considering the average number of links is too rough an indicator to predict the convergence patterns. It must be completed with an analysis of the distribution of the node number of links, because this is the combined influence of this network which can prevent a moderate agent to switch rapidly to extremism. The shortest path relates to the capacity of this network to resist to the influence of one of its members which is beginning to radicalize.



Discussion - conclusion

8.1

The motivation of this paper was to explore the stability of the convergence patterns for different variants of

opinion dynamics model with extremists. The main results are the following:

- Models including interlocutor's uncertainty (RA, GBCU), yield double extreme convergence for high initial uncertainty of the moderates (U) and high proportions of extremists, even when μ is small, and for any type of network. Single extreme convergences are likely for high initial uncertainty of the moderates (U) and small initial proportion of extremists, in relatively small populations and for small values of μ . The lattice structure yields very particular patterns, in which the double extreme convergence takes place as soon as the uncertainty of the moderates is above a critical value, and the single extreme convergence can take place in a very limited part of the parameter space.
- Models without interlocutor's uncertainty (BC, GBC) tend to yield single extreme (or moderate) convergence when the moderates initial uncertainty (U) is above a threshold, for small μ and highly connected, small shortest path networks. In these conditions, the double extreme convergence is limited to a zone between two values of U . For high values of μ , and long path networks (such as lattice for instance), the double extreme convergence takes place for high proportions of extremists.

8.2

These patterns are not significantly modified by the introduction of a noise which is compatible with the general modelling hypotheses.

8.3

The analysis of these differences (and particularly the differences between the convergence patterns obtained on a lattice and the ones obtained on a random network with 4 neighbours on average) led us to identify several explanatory features of these results. One of them seems particularly noticeable:

- The single extreme convergence takes place when the moderates remain clustered and isolated as long as possible from the extremists. In this case, the extremists remain influential, without being influenced by the moderates.
- The clustering of the moderates is favoured by networks where the individuals are all close to each other in the graph.

This observation could be associated to the following interpretation: to remain effective, the extremists must always remain different from the mass of moderates. The big risk for them is to become normalized. This result holds for all the studied model variants.



Acknowledgements

I would like to thank Gérard Weisbuch and Frédéric Amblard for helpful discussions, Bruce Edmonds for raising the question of sensitivity to noise, and the anonymous referee for suggesting to test scale-free networks in order to complete the study about the effect of the shortest path.



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