# Appendix C

Statement 1: In the Metanorms game, assuming continuity and using Axelrod's parameters, there are only two ESSs: one is ($b_i$ = 4/169, $v_i$ = 1 for all $i$) and the other one is ($b_i$ = 1, $v_i$ = 0 for all $i$).

Proof: The argument is similar to the one followed in appendix A. We start by proving that a necessary condition for a state to be an ESS is that every agent is following the same strategy.

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$$\mathrm{Exp}(Payoff_i) = \mathrm{Exp}(Payoff_j) \quad \forall\, i, j \in I\ (m \notin I);\ \forall\, m \in \Theta;\ \forall\, b_m, v_m$$

$$F = \mathrm{Exp}(Payoff_i) - \mathrm{Exp}(Payoff_j) = 0 \quad \forall\, i, j \in I;\ \forall\, m \in \Theta;\ \forall\, b_m, v_m$$

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$$\frac{\partial F}{\partial b_m} = 0 \quad \forall\, i, j \in I;\ \forall\, m \in \Theta;\ \forall\, b_m \in (0,1)$$

$$E \cdot b_m \cdot (v_i - v_j) + \frac{ME}{4} 3b_m^2 \left( v_i \sum_{\substack{k=1 \\ k \neq i,m}}^{n} (1 - v_k) - v_j \sum_{\substack{k=1 \\ k \neq j,m}}^{n} (1 - v_k) \right) = 0$$

$$\forall\, i, j \in I;\ \forall\, m \in \Theta;\ \forall\, b_m \in (0,1)$$

$$E \cdot (v_i - v_j) + \frac{ME}{4} 3b_m \left( v_i \sum_{\substack{k=1 \\ k \neq i,m}}^{n} (1 - v_k) - v_j \sum_{\substack{k=1 \\ k \neq j,m}}^{n} (1 - v_k) \right) = 0$$

$$\forall\, i, j \in I;\ \forall\, m \in \Theta;\ \forall\, b_m \in (0,1)$$

$$E \cdot (v_i - v_j) = 0 \quad \forall\, i, j \in I;\ \forall\, m \in \Theta;\ \forall\, b_m \in (0,1)$$

$$v_i = v_j \quad \forall\, i, j \in I;\ \forall\, m \in \Theta \ \Rightarrow\ v_i = v_j \quad \forall\, i, j \in \Theta$$

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$$\frac{\partial F}{\partial v_m} = 0 \quad \forall\, i, j \in I;\ \forall\, m \in \Theta;\ \forall\, v_m \in (0,1)$$

$$\frac{P}{2} \cdot (b_i^2 - b_j^2) + \frac{ME}{4} \left( v_j \sum_{\substack{k=1 \\ k \neq j,m}}^{n} b_k^3 - v_i \sum_{\substack{k=1 \\ k \neq i,m}}^{n} b_k^3 \right) +$$

$$+ \frac{MP}{4} \left( (1 - v_i) \sum_{\substack{k=1 \\ k \neq i,m}}^{n} b_k^3 - (1 - v_j) \sum_{\substack{k=1 \\ k \neq j,m}}^{n} b_k^3 \right) = 0 \quad \forall\, i, j \in I;\ \forall\, m \in \Theta$$

But we also know that $v_i = v_j = V \ \ \forall\, i, j \in \Theta$ , so we can use that now.

$$\frac{P}{2} \cdot (b_i^2 - b_j^2) + \frac{ME}{4} V (b_i^3 - b_j^3) + \frac{MP}{4} (1 - V)(b_j^3 - b_i^3) = 0 \qquad \forall\, i, j \in \Theta$$

$$2P \cdot (b_i^2 - b_j^2) + (ME \cdot V - MP \cdot (1 - V)) \cdot (b_i^3 - b_j^3) = 0 \qquad \forall\, i, j \in \Theta$$

One solution of the equation above is $b_i = b_j$ . We look for more solutions now.

$$2P \cdot (b_i + b_j) + (ME \cdot V - MP \cdot (1 - V)) \cdot (b_i^2 + b_i \cdot b_j + b_j^2) = 0$$

Considering the feasible range of $V$, $b_i$ , and $b_j$ , the equation above implies $b_i = b_j = 0$, as we show below.

$$2P \cdot (b_i + b_j) + (ME \cdot V - MP \cdot (1 - V)) \cdot (b_i^2 + b_i \cdot b_j + b_j^2) \leq$$

$$\leq 2P \cdot (b_i + b_j) - MP \cdot (b_i^2 + b_i \cdot b_j + b_j^2) =$$

$$= \{P = MP\} = P \cdot (2b_i + 2b_j - b_i^2 - b_i \cdot b_j - b_j^2) < 0 \quad IF\ (b_i \neq 0 \ \text{ OR } \ b_j \neq 0)$$

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$$\therefore \forall\, i, j \in \Theta \ \ (b_i = b_j = 0 \ \text{ OR } \ b_i = b_j) \quad \Rightarrow \quad b_i = b_j \quad \forall\, i, j \in \Theta$$

Therefore we have proved that a necessary condition for ESS is that every agent has the same strategy. Taking that into account it is not difficult to prove that the only three states which fulfil eq. (3) and eq. (4) are

$$b_i = \frac{102131 - 17\sqrt{27808729}}{180360} \approx 0.069 \quad \text{AND} \quad v_i = \frac{29 + 180 \cdot b_i}{1683 \cdot b_i} \approx 0.356 \quad \forall i$$

$$b_i = \frac{4}{169} \approx 0.024 \quad \text{AND} \quad v_i = 1 \quad \forall i$$

$$b_i = 1 \quad \text{AND} \quad v_i = 0 \quad \forall i$$

Of these three states only the last two are ESS since the first one could be invaded by (*e.g.*) a mutant who changed its vengefulness to $v_m = 0$.

Proving that the state where $b_i = 4/169$, $v_i = 1$ for all $i$ is indeed an ESS is tedious but simple. Conditions a) and c) in the definition of ESS are met because every agent is following the same strategy. To prove that condition b) is true we assume that $b_i = 4/169$, $v_i = 1$ for all $i$ except for one potential mutant agent $m$, with boldness $b_m$ and vengefulness $v_m$, and it can be shown that:

$$\text{Exp}(Payoff_m) < \text{Exp}(Payoff_i) \;\; \forall\, b_m, v_m, (b_m \neq 4/169 \;\text{ OR }\; v_m \neq 1)$$

Similarly, we can prove that the state where $b_i = 1$, $v_i = 0$ for all $i$ is ESS.

Assuming $\quad b_i = 1 \quad \text{AND} \quad v_i = 0 \quad \forall i \neq m$

$$\text{Exp}(Payoff_m) = b_m T + (n-1)H + E\frac{v_m}{2}(n-1) + ME\frac{v_m}{4}(n-1)\cdot(n-2)$$

$$\text{Exp}(Payoff_i) = T + (b_m + n - 2)H + \frac{v_m}{2}P + MP\frac{1}{4}\cdot v_m \cdot (n-2) \;\; \forall\, i \neq m$$

$$\text{Exp}(Payoff_m) < \text{Exp}(Payoff_i) \;\; \forall\, b_m, v_m, (b_m \neq 1 \;\text{ OR }\; v_i \neq 0)$$