Abstract

Structurally orientated sociologists tend to neglect the dynamical aspects of social systems, whereas theorists of social systems emphasize systems dynamics but only rarely analyze structural features of their domains. The aim of this paper is to integrate dynamical and structural approaches by means of the analysis of particular artificial systems, namely logical or Boolean networks, and their geometry.

It is well known that the dynamics of Boolean networks and the logically similar cellular automata are governed by control parameters. Less well known is the fact that the geometry of these artificial systems, understood as their topology and metric, also contain specific control parameters. These "geometrical" control parameters can be expressed using graph theoretical concepts such as the density of graphs or geodetical properties. Further, the dynamics of those artificial systems depend on the values for the geometrical parameters.

These mathematical investigations are quite important for social research: On the one hand, social dynamics and social structure appear to be two closely related aspects of social reality; on the other hand, a general hypothesis may be drawn from our results, namely that social structural inequality yields simple dynamics whereas social equality gives rise to complex dynamics. Therefore the dynamical complexity of modern democratic societies may be in part due to their democratic structures.

Keywords:

Boolean networks, social systems, geometry, dynamics, theoretical sociology, control parameters

Introduction

1.1

Though it is rather common to analyze social systems structurally, especially in social network analysis, most researchers do not take into account the connections between the structure of a system and its dynamics. There are many elaborate definitions for the structure of a social system (cf. Freeman 1989), but only a few for social systems dynamics. Only recently have structurally orientated researchers begun to investigate the evolution and dynamics of social networks (e.g.
1.2 In order to overcome this deficiency it will be useful to introduce concepts from geometry and theory of systems dynamics into social research. We define here social systems as spaces of social interactions that can be characterized by specific topological and metrical features. Moreover, it can be shown that the "geometry" of a social system has a particular impact on its dynamics or its evolution respectively. Of course, the geometry of a social system can be changed by specific dynamics; we shall deal with this problem in the last section.

1.3 In another paper (Klüver and Schmidt 1999) we showed that social evolution, defined in the terms of the theory of social differentiation (Habermas 1981; Luhmann 1984), can be described and understood mathematically as the unfolding of dimensions of social spaces. In this paper we shall concentrate on the topology and metric of social systems and their relation to systems dynamics.

Basic definitions

2.1 A social system is a particular domain of social actors whose interactions are governed by specific rules of interaction. The interactions generate consequences, whether intended by the actors or not (Knorr-Cetina 1981). So the dynamics of a social system is simply the sequence of consequences of the interactions. Since the collection of actors, together with their respective relations to other actors, constitute the state of a system at time $t$, the dynamics of a social system consists of the succession of the system's states in time generated by the different rules of interaction.

2.2 The basic concept is that of (social) rules of interactions. It is important to note that we do not favor the classical approach to systems dynamics with this concept. As is well known, mathematical system theorists usually look for differential equations, difference equations or Markov models (e.g. Hanneman 1995) which describe -- and explain -- the system's behavior on a global or macro level (for recent applications of this approach in the social sciences see Epstein 1997). In contrast to this "top down" approach we prefer a "bottom up" approach; that is, the analysis of a system by looking for local interactions and their specific rules. The global behavior of the system, i.e., its dynamics, is an "emergent" property that results from the local interactions.

2.3 We prefer this approach because we believe that the task of sociology is, above all, to analyze and understand social actions in accordance with the classical definition of sociology by Max Weber and others (e.g. Weber 1982). Of course, when one is interested in large-scale social domains it is often necessary to aggregate behaviors and to analyze not individual actors and actions but systems as a whole and their interactions. But even then a bottom up approach is possible and useful if social systems are defined as "collective actors". Social systems are understood only if they are understood as the product of social actions and interactions. Since social actions are governed by particular rules, the task of theoretical sociology may be defined as the analysis of social rules and their consequences, i.e., the dynamics they generate. The task of mathematical (or computational) sociology is, therefore, to construct mathematical models able to deal with social rules. This is one of the main differences between the social and the natural sciences and mathematical sociology must be aware of this difference.
When defining social systems by their rules of interaction a distinction needs to be made between two types of rules. The first type consists of (local) interactions in general, regardless of whether or not elements of the system can interact at all. Let us call this type of rules *general rules* of the system. The second type consists of rules that determine whether or not specific interactions between particular actors or units are possible in the system, i.e., whether or not one or more of the general rules are applicable. This second type of rules we will call *geometrical rules*; one reason for this term is the fact, as we shall show below, that these rules can be represented as the graph of the system.

2.5

An example may help to illustrate this difference. For didactic purposes we constructed a producer-consumer system according to the logic of a predator-prey system but on the basis of a cellular automaton. We did not use the classical approach based on the Lotka-Volterra equations. Instead we introduced different rules of interaction between producers (and sellers) of goods and their prospective buyers (consumers) (see also Epstein and Axtell 1996). The general rules are, roughly: If a producer finds a consumer, then the producer sells some of his goods and the producer gets richer by the transaction and the consumer gets poorer. If a consumer buys goods in several succeeding time steps, he goes bankrupt and cannot consume any more for several time steps. If a producer does not find a consumer for several time steps, he also goes bankrupt, with the number of time steps depending on the number of successful sales in the immediate past. The transformation of consumers into producers is generated at random by a specific probability rate. With these rules, the general interactions between producers and consumers are well defined.

2.6

Yet the general rules do not define which interactions are possible in our artificial market system. So we have to add geometrical rules; that is, rules that determine the conditions when interaction is possible. Since we allow only local interactions we use the well-known geometry of cellular automata. A producer may only sell his goods to a consumer who is in the (Moore) neighborhood of the consumer (the eight adjacent cells) and, vice versa, a consumer can buy only from a producer in his neighborhood. If a producer finds no consumer in his neighborhood, he is able to "look" into the next neighborhood to see if there is a consumer and if so, moves to him. If there is no consumer to be seen, the producer moves at random one step in the grid of the 2-dimensional cellular automaton. Hence a producer can go bankrupt if potential consumers are too far away from him. The geometry of the system is the most important factor for determining whether or not producers can stay in the market. The same kind of argument would apply to the constraints by which physical space limits the possibilities of actual interactions.[2]

2.7

To avoid misunderstanding, it is important to note that the concept of "geometry" refers not to the physical space in which all social interactions take place. Social "spaces" such as institutions, organizations or social networks are constituted by nothing other than social rules of interactions as was often stressed by theoretical sociologists (e.g. Giddens 1984; Habermas 1981). When we speak of "spaces," "geometry," "topology" or "metric" we use mathematical concepts for characterizing particular features of these rules of interactions. A classical and well-known example of geometrical rules can be seen in hierarchical organizations where the rules define explicitly who may or may not interact with whom. Because the general rules define general features of the system's interactions, one may also call them qualitative features. Since the geometrical rules determine how many interactions are possible, the properties they constitute may be called quantitative ones.

2.8

Based on these considerations, it is possible to define a topology for social systems (and dynamical systems in general). We will call two elements (social actors) adjacent if, and only if, the elements can interact immediately. The set of all adjacent elements for a particular element $e$ will be called
the neighborhood of \( e \). As the relation "\( e \) is adjacent to \( f \)" -- \( ad(e, f) \) -- is symmetrical, it follows immediately that the definition of adjacency constitutes a topology for the system.\(^3\) When using graph theoretical representations of a system, this definition implies that a line \textit{without any node between them directly connects} \( e \) \textit{and} \( f \). Note that this definition does not say that the results of the mutual interactions are necessarily the same.

### 2.9

A metric \( d \) for our system is given by the following definition.

**Definition:** For any pair of elements, \( e \) and \( f \), define a \textit{distance measure}, \( d \), by:

1. \( d(e, e) = 0 \)
2. \( d(e, f) = 1 \) if, and only if, \( e \) and \( f \) are adjacent,
3. if \( e \) and \( f \) are not adjacent then \( d(e, f) = n \) if there is a chain of elements \( e_1, e_2, ..., e_{n-1} \) with \( ad(e, e_1), ad(e_1, e_2), ... , ad(e_{n-1}, f) \) all true and \( n \) the smallest length of any such chain from \( e \) to \( f \);
   else
4. \( d(e, f) = m \), with \( m = k + 1 \), where \( k \) is the maximum length for any chain in the system.

### 2.10

It is easy to see that \( d \) defines a metric in the strict sense. The first part of the definition is the first axiom for a metric. The symmetry axiom for a metric follows from the symmetry of the adjacency relation. The triangle inequality, \( d(x, z) \leq d(x, y) + d(y, z) \), for a metric is obviously valid when \( z \) is an element in the shortest chain between \( x \) and \( y \). If \( z \) is not an element of such a chain, then there are two possibilities. First, there is at least one chain connecting \( x \) to \( y \) and \( y \) to \( z \). Then either \( d(x, z) = d(x, y) + d(y, z) \) or \( d(x, z) < d(x, y) + d(y, z) \), since by definition \( d(x, y) \) is the length of the shortest chain between \( x \) and \( y \). Secondly, suppose there is no chain between \( x \) and \( z \). Then there can be no chain between \( y \) and \( z \), hence \( d(x, z) = k + 1 = d(y, z) \) and so \( d(x, z) < d(x, y) + d(y, z) \).

### 2.11

Observe that \( d \) is equivalent to the geodetic coding of the node distance when \( d \) is defined in terms of local interactions.\(^4\)

### 2.12

With these definitions it is possible to speak in a strict mathematical sense of the topology and metric of a dynamical system. From the remarks made above about the two kinds of rules determining the dynamics of a social system, we get the following definition.

**Definition:** The \textit{structure of a social system} is a pair, \((R, G)\), where \( R \) is the set of all general rules of a dynamical system and \( G \) is the set of all geometrical rules - that is, the rules governing the particular topology and metric of the system.

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### Control parameters of dynamical systems

#### 3.1

We mentioned above the sociological importance of basing models of social systems on the concept of rules. Therefore to construct a model for social systems we need to look for formal systems that may be interpreted as rule governed systems.

#### 3.2

Well suited for these purposes are cellular automata (CAs) and Boolean networks (BNs). Logically
they are very similar. Both are logical systems that generate their particular dynamics by local rules of interaction. As they are both potential universal Turing machines (Berlekamp et al. 1982), it is possible to model any system with them that can be modeled by computable functions, as implied by the Church-Turing hypothesis. A rather famous CA is the "Game of Life" by Conway (Berlekamp et al. 1982). A CA is basically a grid of cells that are in different states. CA-rules always have the form "if several adjacent cells of a particular cell are in state x, y, z, ... and if the particular cell is in state w, then the particular cell changes its state to v." For example, "if four adjacent cells of a particular cell (the so called von Neumann neighborhood of this cell) are all in state 1, then the cell's states changes to 0, regardless of the previous state of the cell". The logical simplicity and universal applicability of CAs is without doubt the reason that CAs are used so often for modeling purposes in the social sciences (e.g. Schelling 1971; Klüver 1995; Hegselmann 1996; Nowak and Lewenstein 1996; Passerini and Bahr 1997).

3.3

Boolean networks are not as well known in the social sciences. They are artificial networks whose units are connected by logical, or Boolean, functions; that is, by the familiar functions of propositional calculus. BNs have been investigated by Kauffman (1993), who used them as models for "genetic networks", i.e., models for the genetic controlling of cell differentiation in the organism and for biochemical autocatalysis.

3.4

A simple BN containing three units \(a, b\) and \(c\) illustrates the dynamics of BNs:

\[
\begin{array}{c}
 a & b & c \\
 t & 1 & 1 & 0 \\
t+1 & 0 & 0 & 1 \\
t+2 & 0 & 0 & 0 \\
t+3 & 0 & 0 & 0
\end{array}
\]

The network starts with the initial state, \(a = 1, b = 1\) and \(c = 0\), at time \(t\). The connections between the states for \(a, b\) and \(c\) are all determined by the logical conjunction ("and"). The next state at time \(t+1\) is generated by applying the logical conjunction(s) to the current state. The results follow from the well-known properties of the logical conjunction that "\(a\) and \(b\)" = 1 if and only if \(a = b = 1\). The new value of \(c\) is the result of "\(a\) and \(b\)" = \(c\). Since \(a = b = 1\) at time \(t\), \(c = 1\) at time \(t+1\). Accordingly, \(b = 0\) at time \(t+1\) because this is the result of "\(a\) and \(c\)" at time \(t\), and at time \(t\), \(a = 1\) and \(c = 0\), hence \(b\) must be zero. The same logic implies \(a = 0\) at time \(t+1\). This procedure is iterated and generates the dynamics of our little artificial system. The state, \(a = b = c = 0\), is a point attractor that the system reaches in two time steps. It is easy to demonstrate that the trajectories of this system contain only the point attractors \((1,1,1)\) and \((0,0,0)\), which are always reached in no more than two time steps.

3.5

CAs and BNs are discrete systems that allow for classifying their dynamical complexity by counting the periods of the attractors. When we speak in the following about "simple" or "complex" dynamics we always mean either dynamics which generate trajectories with simple attractors, that is point attractors or attractors with short periods (compared to the number of all possible states), or dynamics with attractors characterized by long periods. For an observer, attractors with very long periods may look like the famous strange attractors of chaos theory; yet it is important to note that finite deterministic systems such as finite CAs or BNs are always periodic with at maximum a "pseudo-strange" attractor of period the length of all possible finite states of the system. The famous "theorem of eternal return" by Poincaré stated this fact a hundred years ago. Of course, for practical purposes such a system behaves like a "true" chaotic system.
3.6 We shall concentrate in the following mainly upon BNs because they are in several aspects more suited for sociological purposes than CAs. Yet the most important results about control parameters are valid for both BNs and CAs.

3.7 Studies on the dynamics of BNs have shown that their dynamical complexity is dependent on the particular control parameters for their rule systems. Kauffman (1993) analyzed the parameter, \( K \), defined as the number of variables in the logical functions of a BN, and determined that \( K = 1 \) yields only simple dynamics with point attractors; \( K = 2 \) (as in our example) generates only trajectories with simple attractors of comparably small periods and \( K = 3 \) yields complex dynamics, i.e., long periods for the attractors with accordingly high sensitivity to changes in the initial states.\[5\]

3.8 Another parameter, discovered by Weissbuch and Derrida (cf. Kauffman 1993), is \( P \), which is the distribution ratio for the values of the units resulting from applying the logical functions. In our example the only logical function is the conjunction function, which distributes the values of 1 and 0 via a truth table as (0,0,0,1), i.e., its result is 0 in three of four cases. Therefore the \( F \)-value of the conjunction function is 0.75. As our system contains only the conjunction function, the \( F \)-value for the whole rule system is also 0.75. Another example is the XOR-function (exclusive or), which distributes the values 0 and 1 uniformly; the XOR-function has the truth values \( a \) XOR \( b = 1 \) if and only if \( a \) and \( b \) have different values. Therefore the truth table of XOR contains the value distribution (1,0,1,0). Accordingly, its \( F \)-value is 0.5. Finally, the logical tautology always generates the value 1 and so its \( F \)-value is 1. In general, \( F \) ranges between 0.5 and 1. The results obtained by Weissbuch and Derrida imply that low \( F \)-values generate complex dynamics and vice versa. By the way, the analogue to the \( F \)-parameter with respect to CAs is the \( l \)-parameter (Langton 1992) that has values in the interval (0,1). \[6\]

3.9 The third important parameter is \( CF \), which is defined as the ratio of "canalyzing functions" to all functions of a BN (Kauffman 1993). The parameter \( CF \) ranges from 0 to 1. A canalyzing function is defined by the property that only one variable determines the result of the function. The conjunction function in our example is a canalyzing function since it suffices to know for \( a \) and \( b \) that if \( a \) has the value 0 then the result is also 0, regardless of the value of \( b \). XOR is not a canalyzing function because one always has to know the values of both variables in order to know the result. For \( K = 2 \) only two of sixteen possible functions are not canalyzing (see endnote 4). The \( CF \) parameter decreases rapidly with an increase in \( K \). The dependence of the dynamics of a BN in regard to canalyzing functions is, roughly: the more canalyzing functions the simpler the dynamics, and vice versa.

3.10 Before we introduce other control parameters two comments are necessary. First, both of the parameters, \( F \) and \( CF \), measure qualities of the general rules, namely the logical functions. Neither \( F \) nor \( CF \) says anything about the geometry of the system and instead they just indicate that with particular values of \( F \) and \( CF \) specific dynamics are probable - which may even be contradictory!. Of course, a particular trajectory always depends on both the rules and the specific initial state. The parameters \( F \) and \( CF \) are "only" statistical parameters insofar as they allow exceptions and can compensate each other. We shall deal with this problem below. The same is also true for the \( Z \)- and the \( l \)-parameter.

3.11 Kauffman was well aware of the fact that the geometry of BNs, which he calls the "wiring structure," may contain other control parameters (1993: 219). As the intention of this article is to
demonstrate dependencies between the geometry of a social system and its dynamics, we have to look for topological and metrical qualities of our systems that may take the role of control parameters. Apparently $K$ is such a parameter as $K$ exhibits topological features of a BN. Yet $K$ is not very well suited for sociological purposes and contains little unique information about a BN (see footnote 5).

3.12

Secondly, so far our considerations have little to do with sociology and the modeling of social systems and instead are a mixture between pure and experimental mathematics and general systems theory. Yet it is easy to see the sociological significance of the control parameters discussed so far and their importance for the analysis of social systems. The parameter $F$ is obviously a measure of the permeability of a system because it determines whether or not the different social states can be reached uniformly by the actors. If we consider, for example, the different states of the units of a BN as different social roles, then it is immediately clear that in caste-like or feudal-like societies admittance to social roles is very restricted. Low social states occur frequently, whereas other social states are less frequent. Hence BNs with high $P$-values can model feudal- and caste-like societies. In contrast, modern societies are characterized by a high social permeability; that is, the actors can, in principle, reach all social roles. Accordingly, BNs with low $P$-values can model the rules. Since many traditional societies exhibit only very simple dynamics, in the sense of the changing of social roles (they are quite static) and modern societies show very complicated dynamics, one sees immediately the significance of the control parameters. They explain mathematically well known facts about the dynamics of different societies.

3.13

The $CF$ parameter plays a similar, important role in decision processes. Consider, for example, the Security Council of the UN, where the permanent members have veto power. The five permanent members may be considered as a logical conjunction with $K = 5$. Only the affirmative vote of all permanent members lets a motion pass. Just one "no" is enough to produce a negative decision. Generally speaking, all hierarchical -- and thus undemocratic decision structures -- can be characterized (we take here no account of abstentions) by canalyzing functions since just one member of the organization decides the final result of a decision process. Democratic decision structures, in contrast, cannot be modeled by canalyzing functions as each vote counts equally. Therefore it should be no surprise that traditional societies with undemocratic decision structures generate only simple dynamics in contrast to modern societies with, in principle, the equality of all members.

3.14

From the viewpoint of a graph representation of a BN -- or a social network - the parameter $K$ is an important topological feature of the system. Yet at least as important is the number of output variables a particular unit is acting on. We have investigated this topological property, which we call $V$ (for the German word "Verknüpfung" = connection). In contrast to $K$, which is measured as an absolute value, we measure $V$ as a deviation from the uniform case where all units act on the same number of units. Then $V = 0$ for the uniform case and $V = 1$ if there is maximum inequality of outputs. Since each unit has $K$ inputs then -- assuming that $K$ is the same throughout the whole net -- maximum inequality means that only $K$ elements have any outputs at all. Thus if the network has $N$ elements, at one extreme all $N$ units have the same number of outputs and at the other extreme, only $K$ elements determine by their outputs the dynamics of the system.7

3.15

We have investigated the influence of $V$ on the dynamics of BNs for the case $K = 2$ but with similar geometries in other respects. Our results are roughly as follows. Values for $V$ equal or close to 0 yield rather complex dynamics. For example, for networks with $N = 16$ there are $2^{16} = 65536$ possible states. With a very low $V$ we often got trajectories with a period of 32,000, that is,
trajectories that include half of the possible states. By increasing the value of $V$, the dynamics get simpler. If $V$ is close to 1 all trajectories have point attractors. Therefore $V$ is unambiguously a control parameter that measures the properties of geometrical rules.

3.16

From a sociological viewpoint $V$ is obviously a measure of the degree of democratic structures of a social system. Democracy is often defined as the formal equality of all society members. Yet formal equality is not enough as a famous aphorism by Anatole France expressed very clearly: "The law with its majestic equality forbids the poor as well as the rich to sleep under the bridges at night." True democracy is only obtained if all society members have the same measure of influence and accordingly the discrepancy between formal equality and unequal measures of influence in bourgeois societies was stressed in particular by Critical Theory, as is well known. So we get again the fact, as with the parameters $P$ and $CF$, that social equality generates complex dynamics and high social inequality is a cause for rather simple dynamics. Consideration of modern and several traditional societies confirms these results, as for example comparisons between feudal medieval Europe and modern bourgeois societies.

3.17

Another topological feature of graphs and networks is the density parameter, $D$; that is, the ratio of existing links between nodes of a graph and all possible links. As a network with $N$ units has $N^2 - N$ possible links, $D$ ranges from $1/(N^2 - N)$ to 1 if we skip the trivial case where $D = 0$, i.e., the network has no links at all. The influence of $D$ on the dynamics of the network is not as unequivocal as that of $V$, but a general trend is clear: The smaller the value of $D$ the more complex is the dynamics of the network, and vice versa. In other words, when more social relations exist between members of a social network, the dynamics of the social system becomes simpler. Again, when we look at modern and traditional societies, we see that traditional social systems such as peasant villages have a high density and very simple social dynamics since they are rather static. Modern societies have comparatively low $D$-values as the people are connected only loosely and social isolation is a common phenomenon. Hence the complex dynamics found in modern societies is no surprise as modern societies can be modeled by formal systems with low $D$-values.

3.18

We have seen that $P$ and $CF$ are measures for the general rules of a system and that $V$ is a measure for the geometrical rules. Another interesting aspect of rules is whether or not they are symmetrical. The concept of a symmetrical rule is derived from the symmetry of relations. A relation, $r(a, b)$, is said to be symmetric if $r(a, b)$ implies $r(b, a)$. By analogy we define an interaction to be symmetric if the statement "a acts via the rule $r$ on $b$" implies "$b$ acts via $r$ -- or another rule $r'$ -- on $a$".

3.19

BNs need not be symmetric and their graphs usually contain many non-symmetric relations. So we think it important, as we discovered, that symmetric BN networks, where all interactions are symmetric, tend to have more complex dynamics than is true for non-symmetric networks. The probability that symmetrical networks generated at random generate complex dynamics is, according to our results, much higher than with non-symmetric BNs. In particular, symmetric networks are rather robust against the influence of other control parameters (see below). Only with rising $CF$, that is, the ratio of canalyzing functions, are symmetric BNs forced into simple dynamics. Therefore we investigated whether symmetry of networks in the sense defined above is itself a control parameter.

3.20

Let $n$ be the number of Boolean functions in a BN and let $k$ be the number of symmetric interactions; that is, interactions between two units $a$ and $b$ such that "if there is a function $f(a,b)$, then there is also a function $g(b,a)$". Then the "symmetry parameter" $Sy$ is defined by:
\[ S_y = k/n. \]

3.21
For the sake of simplicity we investigated \( S_y \) in networks with a small numbers of units (from 7 to 16) that can generate complex dynamics. The results are clear. High values of \( S_y \) generate mainly complex dynamics with the exception of the case mentioned above; i.e., where at least 10 to 15% of the functions are canalyzing functions. The lower the value of \( S_y \) the simpler the dynamics of the corresponding network. \( S_y \) is obviously another control parameter and decrease in the \( S_y \)-value may be interpreted as a breaking up of symmetries. Our results show that low \( S_y \) values generate simple dynamics.

3.22
Now we can understand why \( CF \) has such an influence upon symmetric networks. Consider our network example with three units and logical conjunction, \( f \), as the only Boolean function(s). If one replaces the function \( f(a, b) \) by the XOR-function, \( g \), which is not canalyzing, then there is a local break in symmetry. With the function \( f(a, b) = c \) it is enough to know only the value of \( a \) in order to compute the value of \( c \), whereas in the case of \( g(b, c) = a \) one has to know both values of \( b \) and \( c \) to get the value of \( a \). Although our network example is still symmetric according to our definition, the dynamics of our system is generated by a non-symmetric combination of functions. Accordingly, we found that, in many cases, it is enough to substitute only 10 to 15% non-symmetric interactions into a symmetric network in order to reduce the system to simple dynamics.

3.23
Symmetric relations of interactions also express another feature of equality, namely the equal possibility of influencing the partner(s) of interaction. The famous concept from Habermas (1981) of the "herrschaftsfreier Diskurs" (authority free discourse) illustrates this quite well. Discourses, i.e., communicative interactions, which are free of authority or other social constraints are logically symmetric relations of interactions as the influence of \( a \) upon \( b \) is principally the same as the influence of \( b \) upon \( a \). As these discourses are part of the ideal model of democracy, we get again the fact that democratic, symmetric rules generate complex dynamics while non-symmetric rules, that is rules which express asymmetric distributions of authority among the partners of social interactions, generate simple dynamics.

3.24
Symmetrical rules are, in a sense, a mixture between general rules and geometrical ones. On the one hand, they express general features of the rule system because they assert "if there is an interaction between \( a \) and \( b \), then there is an interaction between \( b \) and \( a \) as well," regardless of whether or not any interaction exists between \( a \) and \( b \). On the other hand, they postulate specific interactions because an interaction between \( a \) and \( b \) necessarily generates an interaction between \( b \) and \( a \). That is why, for example, the number of interactions in symmetric networks is always an even number. Therefore, the fact that inequality in different aspects of a network always tends to generate simple dynamics holds true whether the inequality is measured as a property of general rules, geometrical rules or as a common property of both. Although our research is still going on and the results given in this paper should be taken cautiously, we are convinced that the following "Theorem of Social Inequality" is valid: Theorem of Social Inequality: A social system characterized by a rule system whose properties produce social inequality will generate only simple dynamics; the more the system contains properties of social equality, the more complex the dynamics of the system.

3.25
We saw that all of the control parameters discussed above generate this result, whether it be inequality with regard to the accessibility of social roles (high values of \( P \)), inequality with respect to the power of decision (high proportions of canalyzing functions), inequality with reference to actual possibility of influencing others (high values of \( V \)) or inequality with regard to the equality of
rights in social interactions (symmetric versus not symmetric rules). Therefore the following conclusion from our theorem seems rather obvious. As the different control parameters are measures of different dimensions of (social) inequality, it may be assumed that for each important dimension of inequality there exist a particular parameter that is logically independent of the other parameters insofar as the dimensions of inequality express logically independent aspects of inequality (else it would not be necessary to mention them as specific cases of inequality). In particular, because in social systems there are certainly different and logically independent dimensions of social inequality, it is quite understandable, for these reasons alone, why there are so many different control parameters.

3.26

We mentioned above the fact that the different control parameters can compensate each other. Indeed, with most control parameters, as we found out, it is sufficient to obtain values of just one control parameter for moderately high inequality -- e.g., only average values of $V$ -- to get simple dynamics. Even if the other control parameters have values characteristic for complex dynamics, just one control parameter can compensate the others and "force" the system into simple dynamics. Because networks generated at random have at least one control parameter value in the region for simple dynamics with high probability -- it must be an average value -- we may conclude that social order, that is social systems with point attractors or attractors with very small periods, is mathematically more probable than disorder, i.e. systems with complex behavior. So the old question of theoretical sociology as to why there is social order and not permanent unrest and disorder may be answered in a mathematical way. Order is apparently much more probable than disorder.

3.27

We are well aware that such a mathematical answer is not enough for the "real" sociological meaning of the question. Therefore we take a "Kantian stance" (Kauffman 1992) and vary our answer a bit. The fact that order is mathematically more probable than disorder is a condition for the possibility of social order. Since the behavior of human beings generally follows the laws of probability, we may have here an explanation of the historical fact that periods of social order are much more common than periods of disorder. But this is just a tentative proposal.

Change in social rules

4.1

When we speak of the impact of the geometry of social systems on their dynamics we are neglecting the important fact that social systems not only have dynamics generated by their rules and the particular values of the control parameters as discussed above, they are also adaptive systems in the sense that they can vary their rules according to the demands of their environment (for details see Klüver 1999). The capability of rule changing is also the capability of changing the values of some or all control parameters. Of course, after rule changes the system will most probably shift into simple dynamics again, though the period of rule changing is usually one of great unrest and disorder. It is sufficient that the new rules generate inequality in some dimensions, for then the Theorem of Inequality postulates that order will occur once again. Only rule changes that are directed towards increased equality, that is reforms in the direction of more democracy, will produce more complex dynamics than the system had before the rules changed. Because complex dynamics imply that the system will deviate from favorable states -- their trajectories do not reach an attractor near those states --, systems emphasizing democracy must change their rules more often than societies characterized by inequality.

4.2

We have investigated the (mathematical) conditions favorable to the adaptive success of systems,
i.e., the measures of variability the system rules must have in order to reach desired goals. The main results are that there exist not only control parameters in the sense defined above which control the particular dynamics of the system but also meta parameters, that is measures which control the variations of the system's rules. In this article we can only mention this fact (see Klüver 1999). Yet the question remains as to which rule properties an adaptive (social) system will change when it is forced by environmental pressures to do so. As there are many control parameters; that is, aspects of the rules that the system may change, each adaptive system has to decide which "parameter knob" it will turn first. Of course, each system will mainly introduce changes needed to conserve its particular features. By the way, when we speak of "systems" which will change their rules etc., this phrase is just an abbreviation for the more elaborated expression that of course only social actors "act", "obey or change" social rules and so forth.

4.3

The short history of German reforms in the educational system during the last three decades illustrates this point. At first everything in the old educational system was open to change, including the accessibility of different school levels and in particular the "geometry" of the school system, i.e., the division into three different kind of schools. Yet thirty years later one may observe that change took place primarily in the accessibility of different levels and this goal was reached by increasing the probability for a pupil to reach higher levels. From the viewpoint of systems analysis this may be interpreted as just changing probability values for social, i.e., educational, mobility. Therefore these reforms may be seen as just changing the probability values for social mobility and they left constant the geometrical aspects of the German educational system.

4.4

It may be that an adaptive system always tries first to vary probability values of stochastic rules and to keep everything else as constant as circumstances will allow. For this reason we investigated the influence of stochastic rules on the dynamics of the systems and found out that there is -- at least -- one stochastic control parameter that can be described roughly as follows: the more equal the probability values in the stochastic rules, the more complex are the dynamics of the system, and vice versa. Obviously we have here a confirmation of our Theorem of Social Inequality for the case of stochastic rules. Our conjecture is that social systems, as in the case of rule changes, will try to vary the probability values of stochastic rules first, next the general rules and lastly the geometry of the social system. Perhaps the main difference -- in a mathematical sense -- between reform and revolution is the difference between changing just probability values in the former case and changing the geometry of the system in the latter case. But these are questions for further empirical research, though it can be shown mathematically that changes in probability values will suffice in most cases (Klüver 1999) - reforms, in this sense, are mostly sufficient.

**Conclusion**

5.1

The theoretical basis of our article is the conviction that social reality is to be understood only as the product of social actions and interactions governed by specific social rules. The great theorists of sociology such as Weber, Parsons or Habermas always knew that and that is why the classical methods of the natural sciences only achieved limited success when applies to the social sciences. Therefore one has to look for formal systems which can serve as models of local interactions and then investigate their particular features.

5.2

Results obtained in Artificial Life about CAs and BNs, as well as our own results, demonstrate that it is possible to model social reality adequately in accordance with this theoretical basis of sociology and thereby gain insights into some mathematical regularities of social dynamics. Of course, these
are only first and rather rough initial results, but they are a beginning. Mathematical sociology in a classical sense always had the problem that their means were not elaborate enough to tackle the problems dealt with by the mainstream of sociology. This is probably the reason why mathematical sociology never became part of the core of theoretical sociology (see e.g. Fararo 1997). If we remember the heritage of theoretical sociology, perhaps concepts and methods described in this article can lessen the difference that still exists between mathematical sociology and the main concerns of the sociological community. The mainstream of sociology can be convinced of the importance of formal methods only if the main problems of theoretical sociology can be investigated by these methods. So let us try to do so.

Notes

1 We obviously do not follow the famous definition of Luhmann (1984) that social systems consist of "communications" and not of actors. This definition has its place in Luhmann's theory but is not well suited for the purpose of mathematical analysis.

2 One of the advantages of modeling dynamical systems in this manner is that the geometry of a system can easily be taken into account. This can be seen even more clearly when modeling predator-prey systems where the geometry of the artificial system is the model of the geometry of the physical space. The Lotka-Volterra equations have no regard for the physical space of predator-prey interactions.

3 The verification of this statement is left to the reader. When using the term "topology" we refer to the classical neighborhood axioms of Hausdorff, which can be found in any book on set theory or general topology.

4 Strictly speaking, this definition is only valid for non-directed graphs because in directed graphs there need not be interactions between both a and b both. It may be necessary to introduce a "weak" metric, that is a metric where the axiom of symmetry is not always valid. It may be useful to define the distance, for example, between a superior (sup) and a subordinate (sub), with d(sup, sub) = 1 and d(sub, sup) = 1.5 in order to consider the fact that the influence of the superior to the subordinate is bigger than that of the subordinate to the superior. But these are questions for the future.

5 In contrast to Kauffman we do not believe that K is an important parameter, though the results reported by him are, of course, valid. We think K to be a "derived" or secondary parameter insofar as the simple dynamics for K = 1 or 2 are just a consequence of the fact that in these cases most Boolean functions have high values of P or CF (see below) and vice versa. It is quite easy, by the way, to generate complex dynamics with K = 2. But these are problems for specialists and we shall not develop them further in this paper.

6 For the sake of simplicity we skip the Z-parameter (Wuensche and Lesser 1992), which measures the probability of computing backward; that is, determining a system's state at time t-1 with known state at time t.

7 When K is not equal throughout the net the definition of V is a bit more complicated but basically the same. We skip here the mathematical details for how V is measured. Readers who are interested in these details may contact the authors for further information.

References


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