Abstract

Based on a duopolistic set-up where firms produce software products with respective support packs, we analyze firms' predetermined monitoring and their pricing decisions, as well as the impacts of these factors on welfare. Under presence of end-user piracy, users are classified as support-dependent and support-independent. First, a theoretical model is derived, but, due to its complexity, a numerical example is employed to derive the results. We observe that firms that are in competition face a menu of monitoring and pricing combinations. Our results indicate that (i) firms may use monitoring and pricing as strategic complements, rather than substitutes, (ii) profits are not necessarily an increasing function of both monitoring rates and prices, and welfare improvement from the lowest set of monitoring and pricing levels is possible, (iii) firms may prefer improvement in software rather than support packs, targeting especially the support-independent users.

Keywords:
Duopoly, Monitoring, Pricing Strategy, Software Piracy

Introduction

1.1

Intellectual property has been facing the risk of being reproduced without the authorization of its owner since the beginning of industrialization. Today, technology increases productivity in daily life with use of digital products, but it also eases copying of the same products with almost minimal replication costs.

1.2

Piracy, which is one of the most obvious and harmful examples of intellectual property rights violation, is a phenomenon that affects both developers and users of digital goods significantly. On
these manners, as an example, the Digital Millennium Copyright Act (DMCA) passed in the US in 1998, is initiated to protect developers. Digital rights management (DRM) systems, mostly through technology-based protection, provide protection against infringement (Sundararajan 2004). On the same issue, considering both producers and users, TRIPS Agreement (1994) Article 7 states that "The protection and enforcement of intellectual property rights should contribute to the promotion of technological innovation and to the transfer and dissemination of technology, to the mutual advantage of producers and users of technological knowledge and in a manner conducive to social and economic welfare, and to a balance of rights and obligations" (TRIPS Agreement).

1.3 Under presence of piracy, whereas the tools available to a profit maximizing firm are determining the price and enforcement policies, users try to maximize their expected benefit (utility). So, the role assigned to a governing authority in a digital goods market, although not covered in this paper, is to promote the development of advanced products by providing firms the environment to experience profits, while at the same time to improve consumers' welfare (economic and social welfare in the TRIPS statement above for producers and users of software, respectively).

1.4 Regarding the magnitude of piracy of digital products alone in the software industry, the Business Software Alliance (BSA) formed by software producers claims the following figures: the 2006 worldwide piracy rate is estimated to be around 35% amounting to retail worldwide revenue losses of 40 billion USD. Only in the United States, the total loss of revenue in the same year is estimated to be almost seven billion USD with a 21% piracy rate (BSA 2007a). However, research done by Gayer and Shy (2005: 477) also claims that such estimated losses from piracy may be inflated.

1.5 On copying of digital products, almost all papers in the literature, with exceptions like Shy and Thisse (1999) and Belleflamme and Picard (2007), concentrate on a single-firm setting. The implications of enforcement/monitoring and piracy on welfare in a two firm setting under competition, and how these firms may strategically determine prices and set monitoring levels jointly under presence of piracy, have not been analyzed in detail. Several digital product markets consist of competing firms rather than a single firm operating alone in the market, which is the common assumption of most papers in the literature; digital encyclopedia (Encarta and Britannica), office suites (Microsoft and Corel), econometrics software packages (among many, E-Views and Stata), video game software (among many, Nintendo and Electronic Arts) can be given as examples for markets with competing firms.

1.6 In these digital product markets, in addition to the main product, producers also bundle their products with digital or non-digital support packs in the forms of manuals, on-line help, downloading updates, or playing games on-line with other registered users, targeting especially the users who make use of such support. Hence, for users who value the support pack a pirated copy is an imperfect substitute/inferior/vertically differentiated product (Sundararajan 2004). This is another topic that is not covered extensively in the literature.

1.7 In this paper, first, we set-up a theoretical model of duopoly. Due to the complexity of the model, analytical results are not possible to derive (see the explanation in Section 4). So, benefiting from a numerical example, we look at two firms' simultaneous pricing strategies while incorporating the factors stated above. This paper's contribution to the literature is showing the strategic interaction in a duopoly setting with competition, where firms offer a menu of prices and exercise appropriate monitoring rates (determined jointly by firms), while considering the welfare impacts of piracy. Several hypothesis put forward in the literature are tested with the help of our model. And, lastly, providing an answer to the question on whether firms prefer to invest in
improving quality of software or the bundled support pack is also an extension handled in this paper.

1.8

This paper is organized as follows. In Section 2 we present a literature review and our research questions which are followed by the set-up of our model in Section 3. We summarize the solution procedure of the model and present our propositions in Section 4, and then conclude in Section 5.

Literature and research questions

2.1

The extensive literature on piracy of digital goods is reviewed comprehensively by Peitz and Waelbroeck (2006). Among the several issues on intellectual property protection and piracy of digital products considered in the literature, our paper concentrates on the following themes and their implications on consumers and producers:

Implications of piracy on welfare

2.2

Factors leading to piracy are grouped as being social, economic and institutional. Among others, factors such as price of software and social influences can significantly affect whether a person will pirate, or not (Lau 2007). End-user copying that we concentrate on has also several welfare implications; consumers may benefit from unauthorized reproduction, but as a consequence of decreasing firm profits, incentives for product improvement in the future may also decrease. In his seminal paper Johnson (1985: 159) concludes that, in the short-run, piracy may result in increases in consumer surplus, but also in decreases in firm profits; and, in the long-run, due to disincentives to develop new products, there will be inefficiencies in welfare. The short and long-run implications of piracy are also considered by Bae and Choi (2006) who emphasize reproduction and degradation costs. In the short-run, an increase in reproduction costs results in decreased piracy and more efficient products, but the net effect on social welfare can be both positive and negative. Finally, Peitz and Waelbroeck (2006) state that with network effects and increased installed base size end-user piracy may improve welfare, and recapitulate that welfare implications of piracy are influenced by the characteristics of the particular industry analyzed.

Copyright protection

2.3

Copyright protection can be done by using technological tools or intensive monitoring. Increased copyright protection is claimed to lead to welfare inefficiency due to underproduction and to welfare improvement due to underutilization. Conner and Rumelt (1991) state that increased protection raises both product price and profits. At the same time, the authors conclude that in a market where network externalities are present, increased protection may lead to less profit to the firm and less benefit to the buyer. Yoon (2002) seeks the optimal level of copy protection in his model. Contrary to the general claim, Yoon (2002) argues that an increase in copyright protection may decrease or increase the social welfare loss due to underutilization. On the same issue, Chen and Png (2003) show that after increased detection some copiers may switch to not using the product at all, and, hence, welfare may decrease where producers do not benefit from protection. However, some copiers may also start buying which will result in an increase in profits and welfare.

Tools of protection: Pricing, monitoring, and copy protection
Deterrent or preventive strategies can be exercised against piracy. Gopal and Sanders (1997: 30) point to, as examples, hardware or software based tools as preventive and publishing policy guides and auditing as deterrent controls. Chen and Png (2003) state that mechanisms directly targeting piracy are protection and enforcement. Gopal and Sanders (1997) conclude that profits may decrease with preventive but increase with deterrent controls.

Execution of these mechanisms has different welfare consequences, and, hence, they are not necessarily substitutes. Chen and Png (2003), using a parameter to represent detection in their model, show that social welfare can be improved by reducing prices rather than increasing monitoring rate, while producers prefer to exercise excessive monitoring. In addition, the authors propose limit pricing with sufficiently low prices and high monitoring rates to prevent piracy. Incorporating quality improvement to his analysis, Sundararajan (2004) shows that such an improvement may result in lower prices and profits but also improve welfare by increasing demand. Deterrence of commercial piracy is the subject of Banerjee (2006). Letting the government conduct monitoring and setting the penalty rate, following the incumbent monopolist's pricing strategy, it is derived that no monitoring and entry of a commercial pirate is the social optimal outcome.

Under presence of network externalities, Shy and Thisse (1999) in a two firm setting derive that if the network externality is weak, then firms protect their software and get higher profits. On the other hand, if the network externality is strong enough, prices and profit levels are higher when firms do not protect their software. Pricing in a duopoly setting is also considered by Belleflamme and Picard (2007), but with protection mechanism such as increasing the cost of copying technology and decreasing the value of a copy, rather than monitoring.

Digital or non-digital material which we call the support-pack, leading to increases in benefits from the digital product, can be on-line help, phone support, printed manuals, tutorials, or discounts on upgrades. Support-dependent users value such services bundled to the main product highly, whereas support-independent advanced users do not benefit from them at all. Introduction of such heterogeneity among users can also represent individuals with differing risk-aversion parameters (Shy and Thisse 1999: 168). Another extension of such a differentiation among users is dividing the population into ethical (support-dependent) and non-ethical (support-independent) users, where the latter advanced users may prefer to pirate the digital product (Chen and Png 2003; Liu and Fang 2003).

Research questions

Given the vast number of papers on the economics of illegal use of digital products in the literature, using our model set-up, we particularly concentrate on and test the following hypothesis, and discuss the relevant policy implications:

- Two strategic tools of intellectual property rights protection are pricing and monitoring. Given the jointly pre-determined monitoring levels, what does the introduction of a "competitive two-firm setting" have to add about pricing strategies of these firms? In addition, are the widely discussed tools of monitoring and pricing decisions strategic complements, or substitutes?
- What are the optimal levels of profits and consumers' surplus, and can such levels be
implemented?

- What are the incentives of firms for investing in products or support packs with advance quality? How will social welfare be affected with production of these advanced products?

**The model**

3.1
We develop a model of a software industry under presence of a government. The industry consists of two profit-maximizing firms producing two differentiated software packages. We denote these software packages, produced by firms A and B, as A and B, with the respective prices of $p_A$ and $p_B$. We assume that the products are already developed and production of additional software packages with the same quality is costless for both firms, since the technology enables firms to make identical copies easily and efficiently. However, under presence of piracy, due to availability of CD and DVD writers and peer-to-peer (P2P) file sharing networks, pirated (or cracked) copies are also identical to originals. The only difference between the pirated software package and the original one is the support pack bundled in the original software. The support pack may contain printed manuals, how-to books and technical support via phone or on the Internet for buyers of the original software package.

3.2
Besides Microsoft Office and Corel Office Suite, video game software, or statistics software packages, another application of our model can be the digital encyclopedia market. Britannica and Encarta are similar products regarding content and prices, but provide different support packs like frequency of on-line updates, Encarta's local editions, or Encarta's provision of access to MSN channels and Britannica's provision of access to information updates from magazines like The Economist and Newsweek (Alevizou 2002).

3.3
In the model, we assume that the digital products can be cracked and firms do not use any protective device to prevent piracy, but pirates cannot utilize the benefits of the support pack, which may or may not be in digital form. The extra utility provided by the support pack is denoted by $s_A$ for software package A and by $s_B$ for software package B. We also assume that production of additional support packs with the same quality is costless for both firms.

3.4
Another assumption we make is that the firms' roles besides production are setting the level of monitoring $\mu$ jointly, which mostly takes the form of *ex-parte* searches or criminal raids to identify infringers (in one of these recent police raids, an international company settled to pay 3.5 million USD for using unlicensed software, BSA (2007b)). On this assumption, within an industry, the monitoring rate can also be assumed to be set by an alliance of producers like the BSA. Specifically, we assume that firms commit to a two-step decision process regarding the monitoring levels exercised. In the first step, firms jointly determine the level of monitoring, and, in the second step they apply this level without deviating. This assumption is discussed further in Section 5.

3.5
The government is responsible from determining the exogenous penalty level $f$ that is charged to pirates when piracy is detected. The firms incur the costs associated with monitoring $C(\mu)$, and the government collects the penalty $f$ from infringers caught. This penalty level is a fixed amount independent from the quality of software, or a support pack.

3.6
The software packages are doing similar tasks, but are not perfect substitutes. They are
differentiated in the sense that each software package has some superiority over the other like differences in the design, functionality, or user-friendliness. All of these differences add up to the quality of the software package. The quality of the software packages are denoted by A and B, and are uniquely determined by firms A and B, respectively.

3.7
Firms are located at each end of the unit interval; A is located at zero and B at 1. The value attached to software package A by users is maximized at zero with $A > 0$. Similarly, the value attached to software package B is maximized at 1 with $B > 0$. Hence, a user's preference of a product over the other is directly related with his/her location on the unit interval. Such preferences are formed over factors like the user's habits, familiarity with the interface, etc.

3.8
In the first stage of a three-stage game, the government announces the penalty level charged to pirates caught. In the second stage, after observing the government's policy and deterrence against infringers ($f$ is exogenous to firms), firms (or the alliance like the BSA) jointly set the level of monitoring, determine and announce the qualities of their software packages and support packs, and set their profit-maximizing prices. In the last stage, users decide on either to enter the game by buying or pirating a single unit of software, or not to enter the game and leave with their reservation utility of zero.

3.9
Regarding the users, the support-dependent (ethical) users denoted by $x$, gain extra utility from the support pack provided by software manufacturers (companies, universities, or ethical users that buy licensed products are in this category). They are assumed to be strict buyers and they either buy software, or do not. They are uniformly distributed along the unit interval $[0, 1]$. The second type of users, the support-independent users (or non-ethical users) $y$, do not utilize the support pack. These advanced users do not need the support pack at all and this may make them potential pirates. Based on their expected returns, they may either pirate, buy software, or not buy, and are also uniformly distributed along the unit interval $[0, 1]$. The total number of support-dependent and support-independent users in the model is equal to 2. How many among both support-dependent and support-independent users will purchase or pirate will be determined endogenously in the model based on the prices charged.

3.10
Users have to choose among five options: (i) they can buy software A, (ii) buy software B, (iii) pirate software A, (iv) pirate software B, or (v) do without any software package. All users are assumed to be risk-neutral.

3.11
Compared with other papers in the literature, as done by Chen and Png (2003), we let the firms set the monitoring rate, but differing from the single firm model of Chen and Png (2003), our model consists of two price-setting and profit maximizing firms in the presence of a competitive market structure. We also differ from Shy and Thisse (1999) in the sense that whereas in their and our model the software industry consists of two price-setting firms selling differentiated software packages, Shy and Thisse (1999) investigate the effects of software protection policies under weak and strong network externalities. Their model and our model classify users as being either support-oriented or support-independent, but monitoring and penalty are not included in Shy and Thisse (1999). Yoon (2002) specializes in analyzing underutilization and underproduction, whereas we do not. Bae and Choi (2006) concentrate on a single seller with a single good, and do not analyze effects of different levels of monitoring and penalty level on piracy. The support pack in our model is similar to their degradation cost component, but the authors are interested in the relationship between the two costs of piracy and efficiency in both short and long-run settings. Sundararajan (2004) considers the impact of technological tools on preventing piracy.
Utility profiles of support-dependent users

3.12
Support-dependent users either buy one unit of a software package, or do without it and leave the game with the reservation utility of 0. If a support-dependent user located at \( x \) buys software package \( A \) at price \( p_A \), he/she will get his/her valuation of \((1-x)\) times \( A \) plus a constant utility \( s_A \) from utilizing the support pack yielding the net utility of \((1-x)(A+s_A)-p_A\). Similarly, net utility of a user who buys software \( B \) is given by \( x(B+s_B)-p_B \). We determine the location of a marginal user on the unit interval by setting up the relevant individual rationality and incentive compatibility constraints.

Utility profiles of support-independent users

3.13
Net utility of a support-independent user who prefers to buy software \( A \) is given by \((1-y)A-p_A\). The expected utility received by a support-independent user who pirates software \( A \) is given by the difference of utility received from piracy minus penalty paid if the user is caught as \((1-\mu)(1-y)A-\mu f\). Similarly, the utility of a user who buys software \( B \) is given by \( yB-p_B \), while the expected utility of a user who pirates \( B \) is equal to \((1-\mu)yB-\mu f\).

3.14
We can summarize the linear (expected) utility function for each option as:

\[
U(x, y) = \begin{cases} 
(1-x)(A+s_A)-p_A & \text{if a support-dependent consumer buys } A \\
(x(B+s_B)-p_B & \text{if a support-dependent consumer buys } B \\
(1-y)A-p_A & \text{if a support-independent consumer buys } A \\
(1-\mu)(1-y)A-\mu f & \text{if a support-independent consumer pirates } A \\
yB-p_B & \text{if a support-independent consumer buys } B \\
(1-\mu)yB-\mu f & \text{if a support-independent consumer pirates } B \\
0 & \text{if a consumer does without any software}
\end{cases}
\] (1)

3.15
Again, the location of a support-independent marginal user on the unit interval is determined by setting up the relevant individual rationality and incentive compatibility constraints.

Solving the model

Demand and welfare analysis of support-dependent users

4.1
On the unit interval, since all users located to the left of \( x_{AB} \) (marginal user who is indifferent between buying \( A \) and buying \( B \)) buy software package \( A \), the demand firm \( A \) faces is \( x_{AB} \). Similarly, all users located to the right of \( x_{AB} \) buy software package \( B \), and the corresponding demand for software \( B \) is \( 1-x_{AB} \).

4.2
Depending on whether \( x_{AW} < x_{BW} \) or \( x_{BW} < x_{AW} \), different demand functions will be used in the calculations, where \( x_{AW} \) is the marginal user indifferent between buying \( A \) and doing without a
software, and \( x_{BW} \) is the marginal user indifferent between buying B and doing without.

### 4.3 Support-dependent users

Support-dependent users buying software package A lie in \([0, x_{AB}]\). We sum the expected net benefit of these users over \([0, x_{AB}]\) to get the total consumers' surplus, where we define \( p_A^E \) and \( p_B^E \) as the equilibrium prices of software packages A and B, respectively. Hence, the total surplus of support-dependent users buying software package A and B are calculated by integrating over \([0, D_i^{SD,E}]\), where \( D_i^{SD,E} \) is the expected demand function for A and B. Depending on the prices charged by the producers, relevant demand functions will be used as \( D_A^{SD,E} \) and \( D_B^{SD,E} \) in the integrals.

**Demand and welfare analysis of support-independent users**

### 4.4 Support-independent users

Support-independent users have five alternatives to choose from: Buying A, buying B, pirating A, pirating B, and doing without. We define \( y_{AW} \) as the marginal user indifferent between buying A and doing without. Up to \( y_{AW} \), users buying software package A gain non-negative utility. This implies that the support-independent users buying A must lie in \([0, y_{AW}]\). Similarly, support-independent users pirating software package A must lie in \([0, y_{A'W}]\), where \( y_{A'W} \) represents the users who are indifferent between pirating A or doing without.

### 4.5 Support-independent users

Support-independent users buying software package B lie in \([y_{BW}, 1]\), whereas users pirating software package B lie in \([y_{BW}, 1]\) with \( y_{BW} \) representing the users who are indifferent between pirating B or doing without. These four points \( y_{AW}, y_{BW}, y_{A'W}, \) and \( y_{BW} \) must also lie along the unit interval. Remaining marginal users defined earlier lie between these four points. In the following computations, if any of these four points is calculated to be less than zero, then it is set equal to zero. On the other hand, if any of these four points exceeds 1, then it is set equal to 1. Making these simplifications keep these points in the unit interval without loss of generality.

### 4.6 Demand functions

Each ordering of \( y_{AW}, y_{BW}, y_{A'W}, \) and \( y_{BW} \) gives us different demand functions. These four points can be ordered in 24 different ways. However, if \( y_{A'W} < y_{AW} \), then a support-independent user will never pirate software package A. If \( y_{BW} < y_{BW} \), then a support-independent user will never pirate software package B (the proof is presented in Appendix A.1). We exclude the orderings mentioned in the previous statements and rearrangement results in 14 different orderings which are presented in Appendix A.2.

### 4.7 Demand functions

We define \( D_A^{SI} \) and \( D_B^{SI} \) as the support-independent users' demand for original software packages A and B, and \( R_A \) and \( R_B \) are defined to be the support-independent users' demand for pirated software packages A and B, respectively. Price levels are used in the derivation of the demand functions.

**Total demand estimation corresponding to each case**

### 4.8 Total demand

We obtain total demand for any type of software by adding the support-dependent and the support-independent users’ demand functions. Demand for both software are functions of \( p_A, p_B, A, B, s_A, \) and \( s_B \).
and \( s_B \). Since for each of the 14 orderings stated above we will be facing different values for location of buyers, pirates, and users who opt to stay out, we calculate separate total demand functions by analyzing each case.

4.9 Firm A and firm B maximize their profit functions \( \Pi_A \) and \( \Pi_B \) according to \( p_A \) and \( p_B \), respectively. In our analysis, we assume that sunk costs associated with production of software and support pack do not constitute a major factor in profit calculation and they are excluded from the profit functions. For all the cases, we state that if a firm wants to produce a software or support pack with higher quality, then the related cost components will be \((A'-A)^2/2 \), \((B'-B)^2/2 \) and \((s_A'-s_A)^2/2 \), \((s_B'-s_B)^2/2 \) for the improved software and support packs of \( A' \), \( B' \), \( s_A' \), \( s_B' \), respectively. We take cost of monitoring linearly as \( C(\mu) = \mu \). At the end, firm profits for all cases are defined as:

\[
\Pi_A = p_A D_A - (A'-A)^2/2 - (s_A'-s_A)^2/2 - C(\mu),
\]

and

\[
\Pi_B = p_B D_B - (B'-B)^2/2 - (s_B'-s_B)^2/2 - C(\mu),
\]

where \( A, B, s_A, s_B \) are the initial quality levels of software and support packs of \( A \) and \( B \), respectively. Now, we can get the profit functions by substituting the relevant demand functions into (2) and (3).

4.10 For buyers, we define consumers' surplus as the difference between the value attached to a software package and the price charged for the software package. For users who pirate software, consumers' surplus is defined to be the expected benefit of pirating. We define \( CS_A \) and \( CS_B \) as the total surplus of users buying and pirating software package \( A \) and \( B \), respectively. Social welfare, \( SW \), is the sum of total consumers' surplus and firm profits plus the penalty paid by infringers caught which is collected by the government.

\[
SW = CS_A + CS_B + \Pi_A + \Pi_B + \mu f.
\]

4.11 To show how the calculations for each of the 14 orderings are done, calculations for Case A3 are presented as an example in Appendix A.3.

Model solution with numerical values

4.12 With the model set-up characterized above, it is not possible to calculate the optimal price levels for all the 14 cases analytically, since taking derivatives of profit functions with respect to prices deliver complex first-order condition systems. Similarly, within the analytical framework, derivations that lead to the propositions are not possible, since, depending on the ordering of users, we end-up with non-identical results for the 14 cases. Therefore, we define the value spaces for all parameters and calculate numerically the values that the functions of interest take.

i. First, we set the values of software packages \( A, B \in \{1.0, 1.1\} \), the values of the support packs \( s_A, s_B \in \{0.1, 0.2\} \), and the penalty level \( f = 1 \).

ii. Next, we define the vector of monitoring rates between \([0.15; 0.40]\) increasing by 0.025. As
discussed above, firms predetermine and commit to the monitoring rates.

iii. The software prices $p_A$ and $p_B$ both take values within the domain of $[0.1; 0.7]$.

iv. After generating the matrix of price pair combinations by increasing prices by 0.0025, we merge them with the monitoring rates to set-up the $[p_A, p_B, \mu]$ combinations. The numerical calculations described below are run in Pascal for a total of 638,891 cases ($= 241 (p_A) \times 241 (p_B) \times 11 (\mu)$).

v. Next, we define all the marginal users derived in the previous sections and calculate their values using the $(A, B, s_A, s_B, p_A, p_B, \mu)$ combinations. The subcases can be defined using the theoretical results derived earlier.

vi. Using the parameter combinations, we calculate the marginal users, and by looking at their ordering we find the respective case that fits to that combination. Once we find the case, we look for the subcase (which will be one of the 14 subcases) for that particular combination.

vii. The equilibrium concept used: In our model, since the monitoring rates are assumed to be predetermined, after deciding on the monitoring level, firms engage in price competition. Firms select those price levels that are strategically best-responses to each other and which also maximize their profits. We derive the equilibrium prices charged by the firms as follows: Given each monitoring rate, we start with setting one firm's price at a certain level within the interval $[0.1; 0.7]$ and then look for the responding price move of the second firm. After generating all possible price pairs for both firms, we search for the profit maximizing price combination(s). Since in the following analyses we investigate symmetric parameter values for quality set by both firms, trivially, the strategic best-response prices are also derived to be symmetric. In sum, given the discrete values prices can take, we calculate profits for all possible price levels within the defined domain. By comparing alternative prices and respective profits we determine the pairs that are best-responses (from which firms do not have an incentive to deviate and that lead to the Nash equilibrium levels). At the end of this process we find the optimal levels of $(p_A^E, p_B^E, \mu, f)$, and, finally, calculate the optimal profit, consumers' surplus and social welfare. Finally, we end-up with the outputs used in the following analyses.[2]

Firms produce software and support pack with identical qualities: the "Benchmark Case (BC)"

4.13

<Values used: $A = B = 1.0, s_A = s_B = 0.1, f = 1.0$> Now, we would like to describe the way the variables of interest change with model parameters. Once the calculations described above are done, we observe that, depending on the monitoring rates, the parameter space is divided into four regions. The firms can select, depending on the monitoring rates, low monitoring-high price, high monitoring-low price, or high monitoring-high price combinations.

I. Region 1 is reached when firms set monitoring rates of $\mu \leq 0.15$; when this inequality is satisfied, we observe that all of the support-independent users (50% of total users) pirate the products. So, the numerical examples are solved starting with $\mu = 0.150$, since even at this monitoring rate all of the support-independent users pirate software. Hence, for levels below this rate we will observe the same profit and consumers' surplus levels.

i. Prices: Since the monitoring rates in Region 1 are non-deterrent with only the support-dependent users purchasing the products, considering the incentive compatibility of these users, both firms charge highest possible prices-monopoly levels, to them. So, in this region we have the low monitoring-high price combinations.

ii. Demand and piracy: The total number of users in the model is equal to 2. In this region, the support-dependent users purchase and the support-independent users pirate the products with $D_A = D_B = R_A = R_B = 0.5$. A question that can arise here is whether we can make use of this result and try to provide an explanation to real world
phenomena—even with a highly stylized model and its restrictive assumptions. BSA statistics show that, in 2006, the worldwide weighted average piracy rate was 35% with the median piracy rate of 62% (BSA 2007a). This indicates that in half of the countries analyzed, these countries experienced a 62% piracy rate. Hence, Region 1 of this model can be thought of representing currently exercised monitoring rates.

iii. Profits, CS and SW: Profits are equal to revenue minus cost of developing the products. CS is calculated based on utility derived by both support-dependent and independent users. SW is the sum of firm profits and CS.

II. Region 2 is reached when firms set monitoring rates of $0.150 < \mu \leq 0.325$ with no piracy occurring after $\mu > 0.150$. Hence, selecting a monitoring level in this region suffices to eliminate piracy and all users must buy the product from this point on to get positive utility (or they may not purchase and get their reservation utility of 0). As a remark, as long as no restriction is imposed on the firms regarding which level of monitoring to select, firms do not need to exercise continuing levels of monitoring. Hence, to operate in this region, firms directly predetermine the optimal level of monitoring (i.e., profit maximizing), and there can be a discontinuity between $0.150 < \mu < 0.325$.

i. Prices: With piracy eliminated, firms now compete in prices by charging competitive price levels to attract the support-independent users. That is why we observe price levels lower than those of Region 1. This region is reached with high monitoring and low price combinations.

ii. Demand and piracy: If firms prefer to operate under deterrent levels of monitoring, there is no possibility of piracy, and all users purchase the products with expected positive utility. Hence, $D_A = D_B$ equals 1 and $R_A = R_B$ equals 0.

iii. Profits, CS and SW: Profits are positively and both CS and SW are negatively related with monitoring rates. From a social point of view, in this region, highest CS and SW are obtained at the lower margin when $\mu = 0.175$, and highest profits are obtained at the higher margin when $\mu = 0.325$. At $\mu = 0.250$ profits are identical to those in Region 1, but at this level improvements in CS and SW are possible.

From these observations we can deduce that by increasing monitoring, while taking into account the cost related with it, improvement in SW is still possible by charging lower prices and attracting all users.

III. Region 3 is reached when firms set monitoring rates of $0.35 \leq \mu < 0.375$.

i. Prices: Firms start increasing their prices close to the levels of Region 1. We still need to keep in mind that, to be able to charge such high prices, monitoring rates should be within the interval defined for this region. Now, we have the high monitoring-high price combinations.

ii. Demand and piracy: The consequence of high prices is that some support-independent users cannot afford to buy software, but due to deterrent levels of monitoring, they also cannot pirate (support-dependent users even bought at the prices of Region 1). Demand is slightly less than 1. This is the deadweight loss of monopoly due to underutilization. As pointed out by Yoon (2002), users who do not purchase software at such high prices were willing to buy them at lower price levels.

iii. Profits, CS and SW: Comparing the four regions, highest profits are observed in this region due to high prices and high demand. CS and SW are smaller compared with those of Regions 1 and 2.

IV. Region 4 is reached when firms set monitoring rates of $\mu \geq 0.350$.

i. Prices: As with Region 1, analyzing monitoring rates larger than 0.4 does not contribute to our discussion, since users already face monopoly prices in this region.

ii. Demand and piracy: Due to higher prices, demand is lower than the value in Region 3 with, again, no piracy.

iii. Profits, CS and SW: Profits are lower than those of Region 3 due to lower demand. Both CS and SW are the lowest in this region.
The results of the Benchmark Case (BC) can be summarized in the following proposition.

**Proposition 1.** In the symmetric set-up of the "Benchmark Case," profits are maximized at monitoring rates corresponding to price levels close to monopoly (Region 3), whereas consumers' surplus and social welfare are maximized at monitoring rates corresponding to competitive price levels (Region 2). Firms prefer either low monitoring-high price, or high monitoring-high price combinations, since deviations from monitoring rates of Region 1 first decrease and only then increase profits. From a social welfare maximizing point of view, by deviating from monitoring rates of Region 1 to those of Region 2 at \( \mu = 0.250 \) consumers' surplus and social welfare can be increased without decreasing profits.

4.15

The numerical proof of Proposition 1 is given in Table 1. Figure 1 shows the values taken by different variables depending on the monitoring rates selected.

![Figure 1](image)

**Figure 1.** Values that model variables take depending on the monitoring rates selected: A = 1.1, B = 1.0, \( s_A = s_B = 0.1 \), \( f = 1.0 \)

| \( \mu \) | \( p_A \) | \( p_B \) | \( D_A \) | \( D_B \) | \( R_A \) | \( R_B \) | \( \Pi_A \) | \( \Pi_B \) | Total \( \Pi \) | CS | SW |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.150 | 0.5500 | 0.5500 | 0.5000 | 0.5000 | 0.1250 | 0.1250 | 0.2500 | 0.7875 | 1.1875 |
| 0.175 | 0.2625 | 0.2625 | 1.0000 | 1.0000 | 0.0000 | 0.1250 | 0.0875 | 0.0875 | 0.1750 | 1.0750 | 1.4250 |
| 0.200 | 0.3000 | 0.3000 | 1.0000 | 1.0000 | 0.0000 | 0.0000 | 0.1000 | 0.1000 | 0.2000 | 1.0000 | 1.4000 |
| 0.225 | 0.3375 | 0.3375 | 1.0000 | 1.0000 | 0.0000 | 0.0000 | 0.1125 | 0.1125 | 0.2250 | 0.9250 | 1.3750 |
| 0.250 | 0.3750 | 0.3750 | 1.0000 | 1.0000 | 0.0000 | 0.0000 | 0.1250 | 0.1250 | 0.2500 | 0.8500 | 1.3500 |
| 0.275 | 0.4125 | 0.4125 | 1.0000 | 1.0000 | 0.0000 | 0.0000 | 0.1375 | 0.1375 | 0.2750 | 0.7750 | 1.3250 |
| 0.300 | 0.4500 | 0.4500 | 1.0000 | 1.0000 | 0.0000 | 0.0000 | 0.1500 | 0.1500 | 0.3000 | 0.7000 | 1.3000 |
| 0.325 | 0.4875 | 0.4875 | 1.0000 | 1.0000 | 0.0000 | 0.0000 | 0.1625 | 0.1625 | 0.3250 | 0.6250 | 1.2750 |
| 0.350 | 0.5375 | 0.5375 | 0.9625 | 0.9625 | 0.0000 | 0.0000 | 0.1673 | 0.1673 | 0.3347 | 0.5264 | 1.2111 |
| 0.375 | 0.5750 | 0.5750 | 0.9023 | 0.9023 | 0.0000 | 0.0000 | 0.1438 | 0.1438 | 0.2876 | 0.4540 | 1.1166 |
| 0.400 | 0.5750 | 0.5750 | 0.9023 | 0.9023 | 0.0000 | 0.0000 | 0.1188 | 0.1188 | 0.2376 | 0.4540 | 1.0916 |

**Table 1.** Numerical example results for \( A = B = 1.0 \), \( s_A = s_B = 0.1 \), \( f = 1.0 \)

4.16
In sum, following points in different regions are observed from the BC:

i. Region 1 represents a threshold. Moving away from low monitoring-high price combinations of Region 1, firms exercise deterrent levels of monitoring with lower prices in Region 2. In Region 2, with increases in monitoring rates, prices and profits first decrease from the levels of Region 1, and only then increase. On the opposite, with increasing monitoring rates both CS and SW decrease.

ii. In Region 2, at $\mu = 0.250$, with higher monitoring and lower prices, profit levels of Region 1 are reached but now both CS and SW are higher.

iii. In Region 3, with high monitoring and high prices, profits are the highest with SW larger but CS lower than those of Region 1.

iv. In Region 4, with excess levels of monitoring and prices but decreased demand, profits, CS and SW are lower than those of Region 3.

4.17

Given the stylized model we employ, the results derived above describe how monitoring and pricing decisions influence profits, consumers' surplus and social welfare. A question that may arise is, using the model and the results, can we come up with reasons that can help to explain the currently exercised low monitoring levels? Similarly, if higher profits with increasing monitoring are possible, what are the incentives of firms to still apply low monitoring rates? One argument is that, in practice, currently applied deterrence levels, not only against software piracy but in general, are low. To this end, Polinsky and Shavell (2000:72) state that "This is a reasonable supposition given the limited use of fines that we just noted and the low probabilities of their application. For example, the probability of a tax audit averages only 1.7 percent; when combined with the modest penalties for underpayment, one would predict substantial tax avoidance… Given the ample opportunities that exist for augmenting penalties, as well as the possible desirability of increasing enforcement effort, society probably should raise levels of deterrence in many areas of enforcement." Another argument can be firms' incentive to extend their user base by keeping monitoring rates low and allowing for piracy. In such a way, firms may target higher profits in the future. This reasoning also underlines the significance of network effects which are not included in our model. On importance of such network effects, even though with a different model set-up, Shy and Thisse (1999:163) write that "There is a strategic reason why software firms have followed consumers' desire to drop software protection… We show that when network effects are strong, unprotecting is an equilibrium for a non-cooperative industry."

Both firms offer software with higher quality: "Case 2"

4.18

<Values used: $A = B = 1.1, s_A = s_B = 0.1, f = 1.0> Next, we would like to find an answer to whether firms rather prefer to improve the quality of the software product, or the support-pack. Whereas social welfare is directly influenced by underproduction and underutilization, Yoon (2002) states that increasing copyright protection is expected to (i) increase social welfare due to production of advanced products, and, (ii) decrease social welfare due to restricting piracy. Using our model, we would like to derive the potential impacts of developing advanced products on firms and users. Here, a caution is needed while interpreting our results. Although the costs of improvement in software and support pack are reflected in equations (2) and (3), firms may not be able to improve quality infinitely due to restriction of available technologies. Hence, improvement in quality is expected to happen gradually, rather than with a jump or sudden breakthrough.

4.19

Suppose that firms simultaneously decide to produce software that feature advanced tools and hence have better quality than the products discussed in the BC. Observing that such an improvement does not distort the relationship between monitoring rates and pricing described for the BC, a comparison reveals the following points summarized in Proposition 2.
Proposition 2. When both firms offer software with better quality, compared with the BC, profits, consumers' surplus and social welfare increase at all monitoring levels. The improvement in quality also affects prices charged which are higher than those of the BC across all monitoring levels. Some users pirate both software products at monitoring levels corresponding to competitive prices of Region 2.

4.20

The numerical proof of Proposition 2 is given in Table A.1 (Appendix A.4).

4.21

Table 2 and Figure 2 show, for Case 2, the values taken by different variables compared with those taken in the BC.

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Table 2. Changes in model parameters after improvements in software (Case 2) and support pack (Case 3): Comparison with the Benchmark Case values

Figure 2. Values of total profits, consumers' surplus, and social welfare in Case 2 ($A = B = 1.1$) and Case 3 ($s_A = s_B = 0.2$): Comparison with the Benchmark Case (BC) values

4.22

Comparing the results with those of the BC, we observe the points stated below: Prices at all monitoring levels are higher than those of the BC. However, at some monitoring levels, some
support-independent users have positive expected utility from piracy, and with constant penalty level of the BC, we observe piracy of software in Region 2. So, following the increase in quality, the monitoring rates of Region 2 are no more deterrent across all users. With piracy, non-deterrence results in increased consumers' surplus, but due to higher prices charged, also in increased profits. At the end, social welfare increases across all monitoring levels.

**Firms produce support pack with higher quality: "Case 3"**

4.23

<Values used: $A = B = 1.0$, $s_A = s_B = 0.2$, $f = 1.0$> Imagine that firms decide to increase the value of support pack rather than the software they offer to users. As a remark, a direct comparison of results with Case 2 may be misleading due to different changes in magnitude in quality of software and support pack. Hence, a comparison with the BC is more informative. A firm's decision on improving quality of support pack is only motivated by targeting the profits coming from support-dependent users, since the support-independent users are not interested in support pack, at all.

4.24

The consequences of producing an improved support pack can be summed up in the following proposition.

**Proposition 3.** When both firms increase qualities of support packs they produce, compared with the BC, improvement in quality is not reflected in prices except in Regions 1 and 4. There are increases and decreases in profits at different monitoring levels, but consumers' surplus and social welfare both increase at all monitoring rates with no piracy occurring.

4.25

The numerical proof of Proposition 3 is given in Table A.1 in Appendix A.4.

4.26

A comparison with the BC is summarized in Table 2 and Figure 2, and we observe the points stated below: Firms do not alter the prices charged of the BC except the monopoly levels of Regions 1 and 4. Demand does not change except in Region 4. Interestingly, with no increase in prices and demand but higher cost of producing improved support pack, profits decrease in Regions 2 and 3. Hence, at the lowest set of monitoring rates and monopoly levels of production, firms make higher profits, but not in the remaining regions. Nevertheless, support-dependent users benefit from the improvement, and both consumers' surplus and social welfare increase at all monitoring levels.

4.27

So, what is the incentive of a firm to improve the quality of a support pack? The support-pack is provided by only targeting the support-dependent users; and since, as long as their utility is positive, all support-dependent users purchase software anyway, the improvement cannot be reflected in prices. For support-independent users, such an improvement in support pack quality has no meaning. At the end, improvement in support pack quality does not lead to changes from the equilibrium prices charged of the BC, whereas this was not the case with the improvement in software quality which targeted both the support-dependent and the support-independent users.

4.28

The extension of this result can be that, if information on users' valuation of digital products can be gathered, firms can make use of heterogeneity among users by using price discrimination. This could lead to provision of support-pack with higher quality. However, this would still be a complicated issue, since it is difficult for firms to collect such private information (Chen and Png...
Conclusions and discussion

5.1
In a setting with two firms in price competition that produce differentiated software packages with respective support packs, we have looked at the protection of digital products against piracy. Setting-up a theoretical model first—which cannot be solved analytically due to its complexity, and then using a numerical example we have analyzed the impacts of jointly determined and preset monitoring decisions on pricing, and, consequently, on firm profits, consumers' surplus, and social welfare.

5.2
Concentrating on the themes of monitoring, pricing and improvement in product and support pack quality, we have three conclusions. First, we observe that the parameter space defined over monitoring rates can be divided into several intervals. Within these intervals, firms have a menu of monitoring-price combinations. Whereas low monitoring and high price combinations can be called the prevalent ones, firms may prefer high monitoring-high price combinations that result in higher profits. Hence, pricing and monitoring are not necessarily substitutes to achieve increases in profit; rather, strategically, firms can use them jointly in differing combinations.

5.3
Our second conclusion is that firms' and users' objectives do not necessarily coincide, and, hence, it is difficult to implement the socially optimal level of protection without mechanisms like taxes or subsidies. For profit maximization, firms would like to set high monitoring rates but consumers' surplus and social welfare are maximized at lower monitoring rates. Importantly, profits are not necessarily an increasing function of monitoring rates; this statement is also true for prices charged. We derive that deviations from the set of lowest monitoring rates first decrease and only then increase profits. A welfare improvement, by increasing monitoring to deterrent levels, while taking into account the cost related with it and not decreasing profits, is possible by charging lower prices and attracting all users. Research claims that there is room for increases in deterrence levels.

5.4
Finally, our third conclusion is that firms prefer to provide higher quality software rather than higher quality support pack under the assumption of similar cost structure related with improvement. Whereas only the support-dependent users benefit from improved support-pack, all users benefit from improved software. Even though the improvement in software encourages piracy, still, profits and consumers' surplus both increase. To the contrary, for certain monitoring rates, improvement in support-pack quality can even result in decreases in firm profits.

5.5
With respect to caveats, our results are derived based on specific assumptions and a stylized model. Hence, the robustness of the results critically depends on values the model parameters take. The natural question is to what extent do these assumptions influence the results? Regarding the assumption of the same number of support-dependent and independent users, with a larger proportion of support-dependent users in the population, firms may look into improvements in support pack quality to attract users. An extension of the model related with this issue would be incorporating product differentiation and price discrimination. Relaxing the assumption of symmetry in quality of software or support-pack produced would also provide interesting results, albeit with a more complicated model. Incorporating network effects will also help to describe firm behavior when competing firms consider extending their user base while determining pricing and monitoring levels under piracy. We hypothesize that, actually, such effects can be the main reason of exercising low monitoring rates.
Further research can also look at enforcement when firms set their enforcement policies and monitoring rates independently. Welfare impacts of such an analysis may shed more light on enforcement strategies in a multi-firm setting. Relatedly, mechanisms, such as taxes or subsidies, designed to achieve the socially optimal level of welfare may be incorporated into the setting described above. The penalty level is determined exogenously in our model, and this assumption may also be relaxed with a more comprehensive model. Another extension may be incorporating open-source software into the model by considering producers’ or contributors’ incentives to do so, and then, look at the impacts of enforcement versus free software and their welfare effects. In this model, this was not possible with firms targeting profit maximization.

Acknowledgements

The authors thank the editor and three anonymous reviewers for the very insightful comments and highly useful suggestions. We also thank Fikret Adaman, Ayse Mumcu, and İsmail Saglam for helpful discussions. The usual disclaimer applies.

Notes

1. A technical version of this paper with all derivations and proofs is available from the authors upon request.

2. The computer code is available from the authors upon request.

Appendix

Proof of when support-independent users will not pirate

A.1

The marginal user $y_{AW}$ who is indifferent between buying A and doing without is given from $(1-y)A-p_A \geq 0$ as

$$y_{AW} = (A-p_A)/A.$$  \hfill (A.1)

By using the individual rationality constraint, $(1-\mu)(1-y)A-\mu f \geq 0$, we can derive the location of the marginal user $y_{A'W}$ who is indifferent between pirating A and doing without as

$$y_{A'W} = 1-\mu f/(1-\mu)A.$$ \hfill (A.2)

Suppose that $y_{A'W} < y_{AW}$. Then, from (A.1) and (A.2) we can derive that

$$1-\mu f/(1-\mu)A < (A-p_A)/A,$$

which is equal to $\mu f/(1-\mu) > p_A$. \hfill (A.3)

For some support-independent users to pirate software A, we need to have $(1-\mu)A-\mu f \geq A-p_A > 0$ (y is dropped here, but this does not affect the calculations). And once this equation is solved, we get
When we combine (A.3) and (A.4) we obtain
\[ \mu f(1-\mu) > p_A \geq \mu(A+f). \]  
(A.5)

And this implies that
\[ \mu f(1-\mu) > \mu (A+f). \]  
(A.6)

This contradicts our assumption about \((1-\mu)A-\mu f \geq 0\). Hence, no support independent user pirates software A if \(y_{A'W} < y_{AW}\).

**Ordering of marginal users**

\[ A.2 \]

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**Calculations for Case A3.**

\[ A.3 \]

In Case A we have the ordering of marginal types as \(y_{B'W} < y_{BW} < y_{AW}\). If firm B charges a price defined as \(p_B \leq \mu(y_B+f)\), for the subcase \(y_{B'B} \leq y_{AB}\) (which implies that \(y_{B'W} \leq y_{B'B} \leq y_{BW}\)) we can calculate the expected utility of each user by making use of Figure A.1. It can be shown that if firm B charges a price that satisfies \(p_B > \mu(y_B+f)\), no support independent user buys software package B at that price.
In Figure A.1, we observe that at every point up to $y_{AW}$, buying A brings strictly more expected benefit than pirating A. Therefore, in this subcase, no support-independent user will pirate software package A. In order to find the demand functions, we trace the utility curves from zero to 1 and pick up the utility curve that brings more net expected benefit than the others. Starting from zero up to $y_{AB}$, buying A brings the highest benefit, since its utility curve lies above all of the remaining utility curves. From $y_{AB}$ to 1, buying B brings the highest benefit, since its utility curve lies above all of the other utility curves. Thus, in this subcase, support-independent users buy either software package A or software package B, and they never pirate. This implies that with no piracy we have $R_A = R_B = 0$. Support-independent users' demands for original software packages are given as $D_{A}^{SI} = y_{AB} = (A+p_B-p_A)/(A+B)$ and $D_{B}^{SI} = 1-y_{AB} = (B-p_B+p_A)/(A+B)$. Support-dependent users' demand for original software packages are given as $D_{A}^{SD} = (A-p_A+p_B+s_A)/(A+s_A+B+s_B)$, and $D_{B}^{SD} = (B-p_B+p_A+s_B)/(A+s_A+B+s_B)$. After adding up these values we obtain the demand functions for both products. Profits are calculated based on equations (2) and (3). For users who pirate software, consumers' surplus is defined to be the expected benefit of pirating. We define $CS_A$ as the total surplus of users buying and pirating software package A and calculate it as

$$CS_A = \int_0^{\frac{D_A^{\infty,\varepsilon}}{\xi}} [(1-x)(A+s_A)-p_{A,\varepsilon}] dx + \int_0^{\frac{D_{B,\varepsilon}}{\xi}} [(1-y)A-p_{A,\varepsilon}] dy$$

$$+ \int_0^{\frac{D_{B,\varepsilon}}{\xi}} [(1-\mu)(1-y)A-\mu f] dy.$$  \hspace{1cm} (A.7)

Since we have $R_A = 0$ for this case, we end up with
A similar calculation is done for CS_B. Finally, SW for this subcase is calculated using (4).

**Numerical proofs for Propositions 2 and 3.**

A.4

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**Table A.1.** Numerical example results for A = B = 1.1, s_A = s_B = 0.1, f = 1.0 and A = B = 1.0, s_A = s_B = 0.2, f = 1.0

**References**


BSA (2007a) "Fourth Annual BSA and IDC Global Software Piracy Study"
http://w3.bsa.org/globalstudy/.


